Iterative Multistage Soft Decoding of Multilevel Reed-Solomon Codes

Xin Huang †, Li Chen ‡
† School of Electronics and Information Technology, Sun Yat-sen University, Guangzhou, China, 510006
‡ School of Electronics and Communication Engineering, Sun Yat-sen University, Guangzhou, China, 510006
Email: huangx259@mail2.sysu.edu.cn, chenli55@mail.sysu.edu.cn

***Abstract*—This paper proposes an iterative multistage soft decoding (IMSD) for multilevel Reed-Solomon (MRS) codes, achieving both high decoding performance and transmission spectrum efficiency. The proposed IMSD algorithm performs soft-in soft-out (SISO) decoding of each level RS code in a multistage mechanism. The RS decoding is realized by cascading the adaptive belief propagation (ABP) algorithm that produces the extrinsic probabilities for the coded bits and the Berlekamp-Massey (BM) algorithm that estimates the message. The earlier decoding outputs help a later one by providing better *a priori* information for its decoding. Armed with the IMSD algorithm, the MRS codes are designed by analyzing the equivalent channel capacity of each coded level, leading to the heterogeneous structure for MRS codes. Our simulation results demonstrate the performance advantages of the IMSD algorithm as well as the designed MRS codes.**

I. Introduction

Multilevel coding [1], introduced by Imai and Hirakawa, integrates multiple error-correction codes and a high-order modulation. The error-correction codes are called the component codes and they determine the transmitted symbols. For multilevel codes, decoding can be performed in a multistage mechanism which implies level-by-level message recovery. Consequently, a later decoding can be benefited from the outputs of the earlier ones. To exploit the decoding potential, iterative multistage decoding (IMD) of multilevel codes has been proposed by Martin *et al.* [2] and Isaka *et al.* [3]. With the iterative decoding, a multilevel code design using the extrinsic information transfer function has been considered in [4]. The above multilevel codes employ the binary channel codes, such as turbo codes and convolutional codes, as their component codes. In comparison, Reed-Solomon (RS) codes are nonbinary maximum distance separable codes. They have a competent burst error correction capability, making them popular in data communications and storage systems. Therefore, multilevel RS (MRS) codes are important coded modulation schemes [5] [6]. However, the existing decoding algorithms for MRS codes are hard-decision oriented with limited performance. The iterative multistage soft decoding (IMSD) for MRS codes has not been developed. This is mainly due to the difficulty in realizing the soft-in soft-out (SISO) decoding for RS codes. Among the existing RS SISO decoding approaches [7]–[9], the adaptive belief propagation (ABP) algorithm [9] produces the extrinsic information of the coded bits with a polynomial-time complexity, making the IMSD of MRS codes feasible.

This paper proposes an IMSD algorithm for MRS codes, where the RS SISO decoding is realized by the ABP algorithm. The Berlekamp-Massey (BM) algorithm [10] further estimates the message based on the *a posteriori* probabilities provided by the ABP algorithm. In the IMSD scheme, the earlier decoding outputs help provide better *a priori* information for the later decoding, improving the decoding performance significantly. Iterating the extrinsic information of RS coded bits, the IMSD algorithm is able to achieve remarkable iterative decoding gains. We further investigate the design of MRS codes by analyzing the equivalent channel capacity of each coded level, leading to the heterogeneous structure for MRS codes that achieves a substantial performance improvement. Merits of our proposals will be verified by simulations.

II. MRS codes

Fig. 1 shows the structure of an MRS code. It integrates m RS codes and an M-ary modulation, where $m = \log_2 M$. Every m RS coded bits that are respectively produced by the m component codes are mapped in an M-ary constellation symbol.

Let $\mathbb{F}_{2^p} = \{0, 1, \alpha^1, \alpha^2, \ldots, \alpha^{2^p-2}\}$ denote a finite field of size 2^p, where p is a positive integer and α is a primitive element of the field. In an MRS coding scheme, an (n_t, k_t) RS code defined over $\mathbb{F}_{2^p}$ is utilized at level t ($t = 0, 1, \ldots, m-1$), where $n_t = 2^p - 1$ and k_t are the length and dimension of the codeword, respectively. Using the BM algorithm, it can correct at most $\lfloor \frac{n_t - k_t}{2} \rfloor$ symbol errors. Since the RS codes of all levels maintain the same length, let $n = n_t$, $\forall\, t$. Generator polynomial of the (n, k_t) RS code can be defined as

$$g_t(x) = (x - \alpha^1)(x - \alpha^2)\cdots(x - \alpha^{n-k_t}). \tag{1}$$

At level t, given a message vector as $\underline{U}^{(t)} = [U_0^{(t)}, U_1^{(t)}, \ldots, U_{k_t-1}^{(t)}] \in \mathbb{F}_{2^p}^{k_t}$, the message polynomial $U^{(t)}(x)$ is

$$U^{(t)}(x) = U_0^{(t)} + U_1^{(t)}x + \cdots + U_{k_t-1}^{(t)}x^{k_t-1}. \tag{2}$$

The codeword can be generated by

$$\begin{aligned} C^{(t)}(x) &= x^{n-k_t}U^{(t)}(x) + x^{n-k_t}U^{(t)}(x) \bmod g_t(x) \\ &= C_0^{(t)} + C_1^{(t)}x + \cdots + C_{n-1}^{(t)}x^{n-1}. \end{aligned} \tag{3}$$

The codeword is $\underline{C}^{(t)} = [C_0^{(t)}, C_1^{(t)}, \ldots, C_{n-1}^{(t)}] \in \mathbb{F}_{2^p}^n$. For an MRS code, if $k_0 = k_1 = \cdots = k_{m-1}$, all levels exhibit a homogeneous structure. It is therefore called the homogeneous

978-1-5386-3599-5/18/$31.00©2018 IEEE

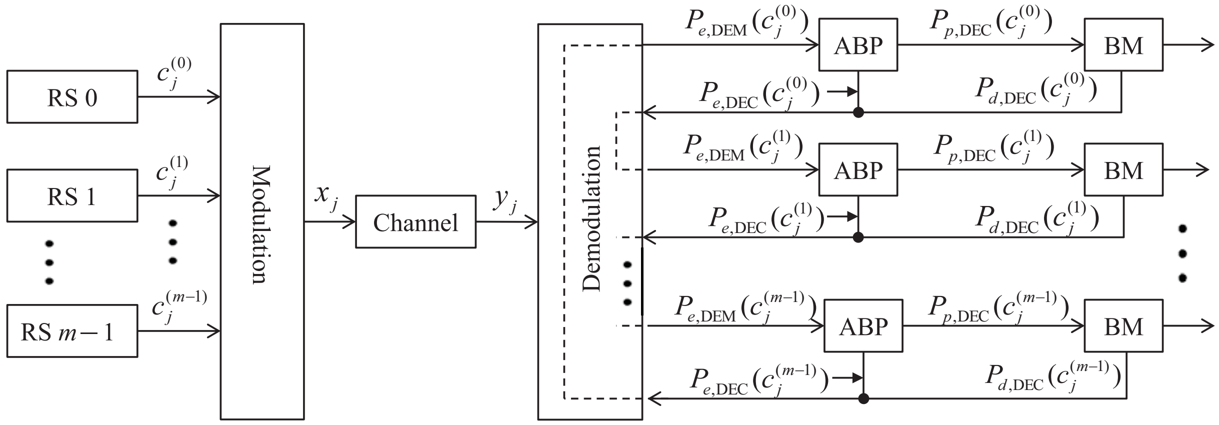

Fig. 1. MRS code and the IMSD scheme.

MRS (HoMRS) code. Otherwise, it is called the heterogeneous MRS (HeMRS) code. Let

$$\mathcal{R}^{(t)} = \frac{k_t}{n} \tag{4}$$

denote the rate of the level t RS code. The rate of the MRS code is

$$\mathcal{R} = \frac{1}{m}\sum_{t=0}^{m-1}\mathcal{R}^{(t)}. \tag{5}$$

For HoMRS codes, $\mathcal{R}^{(t)} = \mathcal{R}$, $\forall\ t$.

For an (n, k_t) RS code, its parity-check matrix $\mathbf{H}_t$ can be defined as

$$\mathbf{H}_t = \begin{bmatrix} 1 & \alpha & \cdots & \alpha^{n-1} \\ 1 & \alpha^2 & \cdots & \alpha^{2(n-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha^{n-k_t} & \cdots & \alpha^{(n-k_t)(n-1)} \end{bmatrix}. \tag{6}$$

Let $\mathbf{A}$ further denote the companion matrix of $\mathbb{F}_{2^p}$, which is a $p \times p$ binary matrix. The binary parity-check matrix $\mathcal{H}_t$ of the RS code can be generated by replacing the entries α^ℓ of $\mathbf{H}_t$ by $\mathbf{A}^\ell$, where $\ell = 0, 1, \ldots, 2^p - 2$.

Given an M-ary constellation symbol set $\mathcal{S} = \{s_0, s_1, \ldots, s_{M-1}\}$, symbol s_i $(i = 0, 1, \ldots, M-1)$ is mapped from a bit array $[\theta_0, \theta_1, \ldots, \theta_{m-1}]$, where $\theta_t \in \{0, 1\}$. In order to map the RS codewords into constellation symbols, each codeword $\underline{C}^{(t)}$ needs to be converted into its binary version $\underline{c}^{(t)} = [c_0^{(t)}, c_1^{(t)}, \ldots, c_{np-1}^{(t)}]$. Let $\mathcal{M}(\cdot)$ and $\mathcal{M}^{-1}(\cdot)$ denote the mapping and demapping functions, respectively, such that $\mathcal{M}(\theta_0, \theta_1, \ldots, \theta_{m-1}) = s_i$ and $\mathcal{M}^{-1}(s_i) = [\theta_0, \theta_1, \ldots, \theta_{m-1}]$. We further denote $[\mathcal{M}^{-1}(s_i)]_t = \theta_t$. Therefore, the m RS coded bit sequences are mapped into M-ary constellation symbols by $x_j = \mathcal{M}(c_j^{(0)}, c_j^{(1)}, \ldots, c_j^{(m-1)})$, resulting in the modulated symbols $x_0, x_1, \ldots, x_{np-1}$, where $x_j \in \mathcal{S}$ and $j = 0, 1, \ldots, np-1$. After the channel, the received symbols are $y_0, y_1, \ldots, y_{np-1}$.

III. Iterative multistage soft decoding

The proposed IMSD scheme for MRS codes is also shown in Fig. 1. Let $P_{a,\mathrm{DEM}}(c_j^{(t)})$, $P_{e,\mathrm{DEM}}(c_j^{(t)})$ and $P_{p,\mathrm{DEM}}(c_j^{(t)})$ denote the *a priori*, the extrinsic and the *a posteriori* probabilities of $c_j^{(t)}$ produced by the demapper, respectively. Similarly, let $P_{a,\mathrm{DEC}}(c_j^{(t)})$, $P_{e,\mathrm{DEC}}(c_j^{(t)})$ and $P_{p,\mathrm{DEC}}(c_j^{(t)})$ denote the *a priori*, the extrinsic and the *a posteriori* probabilities of $c_j^{(t)}$ produced by the decoder, respectively.

At level t, the RS decoding consists of two stages. The ABP decoding yields both the $P_{e,\mathrm{DEC}}(c_j^{(t)})$ and $P_{p,\mathrm{DEC}}(c_j^{(t)})$, where hard decision of $c_j^{(t)}$ will be made based on $P_{p,\mathrm{DEC}}(c_j^{(t)})$. With a hard-decision received word, the BM algorithm decodes the message. If the BM decoding succeeds, we obtain the coded bits' estimations as $\hat{c}_0^{(t)}, \hat{c}_1^{(t)}, \ldots, \hat{c}_{np-1}^{(t)}$. The decoder will feed back the deterministic probability $P_{d,\mathrm{DEC}}(c_j^{(t)})$, where $P_{d,\mathrm{DEC}}(c_j^{(t)} = \hat{c}_j^{(t)}) = 1$ and $P_{d,\mathrm{DEC}}(c_j^{(t)} = 1 - \hat{c}_j^{(t)}) = 0$. Otherwise, the extrinsic probability $P_{e,\mathrm{DEC}}(c_j^{(t)})$ will be fed back. The decoding feedback provides *a priori* information for the next level demapping. At the level $m-1$, $P_{d,\mathrm{DEC}}(c_j^{(m-1)})$ (or $P_{e,\mathrm{DEC}}(c_j^{(m-1)})$) will be fed back for the demapping of level 0. The next round multistage demapping and decoding begin. This iterative demapping-decoding terminates when codes of all levels are decoded or a predefined maximum number of iterations (denoted as N_{GLO}) is reached.

A. SISO Demapping

The demapping and decoding are performed successively for each level RS code. In general, for level t RS code, $P_{p,\mathrm{DEM}}(c_j^{(t)})$ is determined by

$$P_{p,\mathrm{DEM}}(c_j^{(t)} = \theta) = \sum_{s_i \in \mathcal{S}_{t,\theta}} P(y_j \mid s_i) P(s_i), \tag{7}$$

where $P(y_j \mid s_i)$ is the channel observation and $\mathcal{S}_{t,\theta} = \{s_i \mid [\mathcal{M}^{-1}(s_i)]_t = \theta \text{ and } s_i \in \mathcal{S}\}$. The symbol probability $P(s_i)$

is defined as

$$P(s_i) = \prod_{t=0}^{m-1} P_{a,\text{DEM}}(c_j^{(t)} = [\mathcal{M}^{-1}(s_i)]_t). \quad (8)$$

Note that at the beginning, $P_{a,\text{DEM}}(c_j^{(t)} = 0) = P_{a,\text{DEM}}(c_j^{(t)} = 1) = 0.5$, $\forall\ (j,t)$. In order to constitute an iterative demapping-decoding mechanism, the coded bit's extrinsic probability is needed, which can be determined by

$$P_{e,\text{DEM}}(c_j^{(t)}) = \frac{P_{p,\text{DEM}}(c_j^{(t)})}{P_{a,\text{DEM}}(c_j^{(t)})}. \quad (9)$$

Based on (7) - (9), $P_{e,\text{DEM}}(c_j^{(t)})$ can be determined by

$$P_{e,\text{DEM}}(c_j^{(t)} = \theta) = \sum_{s_i \in \mathcal{S}_{t,\theta}} P(y_j \mid s_i) \prod_{t'=0, t' \neq t}^{m-1} P_{a,\text{DEM}}(c_j^{(t')} = [\mathcal{M}^{-1}(s_i)]_{t'}). \quad (10)$$

$P_{e,\text{DEM}}(c_j^{(t)})$ will be utilized by the following RS decoding.

B. SISO Decoding

For the level t RS code, its SISO decoding starts with the following mapping

$$P_{e,\text{DEM}}(c_j^{(t)}) \mapsto P_{a,\text{DEC}}(c_j^{(t)}). \quad (11)$$

The *a priori* log-likelihood ratio (LLR) of $c_j^{(t)}$ can be determined by

$$L_a(c_j^{(t)}) = \ln \frac{P_{a,\text{DEC}}(c_j^{(t)} = 0)}{P_{a,\text{DEC}}(c_j^{(t)} = 1)}. \quad (12)$$

Subsequently, the *a priori* LLR vector of level t RS code is

$$\underline{L}_a^{(t)} = [L_a(c_0^{(t)}), L_a(c_1^{(t)}), \ldots, L_a(c_{np-1}^{(t)})]. \quad (13)$$

Entries of $\underline{L}_a^{(t)}$ will be sorted based on their magnitudes $|L_a(c_j^{(t)})|$, resulting in a refreshed index sequence $j_0, j_1, \ldots, j_{(n-k_t)p-1}, \ldots, j_{np-1}$. It indicates $|L_a(c_{j_0}^{(t)})| < |L_a(c_{j_1}^{(t)})| < \cdots < |L_a(c_{j_{(n-k_t)p-1}}^{(t)})| < \cdots < |L_a(c_{j_{np-1}}^{(t)})|$. Bits $c_{j_0}^{(t)}, c_{j_1}^{(t)}, \ldots, c_{j_{(n-k_t)p-1}}^{(t)}$ are considered as the $(n-k_t)p$ least reliable bits, where their indices are collected in $\Theta_t = \{j_0, j_1, \ldots, j_{(n-k_t)p-1}\}$.

In order to perform the BP decoding, Gaussian elimination is needed to reduce the density of $\mathcal{H}_t$. This will be carried out based on the above sorting outcome. In $\mathcal{H}_t$, columns that are indexed by Θ_t will be reduced to weight one, forming an $(n-k_t)p \times (n-k_t)p$ identity submatrix. Doing so, we cannot only reduce the density of $\mathcal{H}_t$, but also minimize the propagation of the unreliable information during the BP process. Note that it is not guaranteed all columns that are indexed by Θ_t can be reduced to weight one. In this case, we will reduce columns $j_{(n-k_t)p}, j_{(n-k_t)p+1}$ and etc. The above process results in the adapted parity-check matrix $\mathcal{H}'_t$.

The following BP decoding will be performed based on $\mathcal{H}'_t$. Let h_{vj} denote the entry of $\mathcal{H}'_t$. We define

$$\mathbf{V}(j) = \{v \mid h_{vj} = 1, \forall\ 0 \leq v \leq (n-k_t)p - 1\}, \quad (14)$$

$$\mathbf{J}(v) = \{j \mid h_{vj} = 1, \forall\ 0 \leq j \leq np - 1\}. \quad (15)$$

The extrinsic LLR of $c_j^{(t)}$ is determined by

$$L_e(c_j^{(t)}) = \sum_{v \in \mathbf{V}(j)} 2 \tanh^{-1} \left(\prod_{j' \in \mathbf{J}(v) \setminus j} \tanh \left(\frac{L_a(c_{j'}^{(t)})}{2} \right) \right). \quad (16)$$

The *a posteriori* LLR of $c_j^{(t)}$ is further determined by

$$L_p(c_j^{(t)}) = L_a(c_j^{(t)}) + \eta L_e(c_j^{(t)}), \quad (17)$$

where $\eta \in (0,1)$ is a damping factor. Note that matrix $\mathcal{H}'_t$ remains dense for the BP decoding. It still contains many short circles that affect the reliability of the extrinsic information. Hence, η is needed to downgrade the extrinsic influence.

All RS coded bits can be estimated based on the *a posteriori* LLR vector

$$\underline{L}_p^{(t)} = [L_p(c_0^{(t)}), L_p(c_1^{(t)}), \ldots, L_p(c_{np-1}^{(t)})]. \quad (18)$$

The BM decoding is further performed to estimate the message (and codeword). If the BM decoding succeeds, $\hat{c}_j^{(t)}$ can be obtained and $P_{d,\text{DEC}}(c_j^{(t)})$ will be fed back to update $P_{a,\text{DEM}}(c_j^{(t)})$ by

$$P_{d,\text{DEC}}(c_j^{(t)}) \mapsto P_{a,\text{DEM}}(c_j^{(t)}). \quad (19)$$

Otherwise, $P_{e,\text{DEC}}(c_j^{(t)})$ will be fed back to update $P_{a,\text{DEM}}(c_j^{(t)})$ by

$$P_{e,\text{DEC}}(c_j^{(t)}) \mapsto P_{a,\text{DEM}}(c_j^{(t)}). \quad (20)$$

Note that $P_{e,\text{DEC}}(c_j^{(t)})$ can be obtained by

$$P_{e,\text{DEC}}(c_j^{(t)} = 0) = \frac{1}{1 + e^{-L_e(c_j^{(t)})}}, \quad (21)$$

$$P_{e,\text{DEC}}(c_j^{(t)} = 1) = \frac{1}{1 + e^{L_e(c_j^{(t)})}}. \quad (22)$$

The updated $P_{a,\text{DEM}}(c_j^{(0)}), P_{a,\text{DEM}}(c_j^{(1)}), \ldots, P_{a,\text{DEM}}(c_j^{(t)})$ are used to calculate $P_{e,\text{DEM}}(c_j^{(t+1)})$ using (10) for next level decoding.

Note that the ABP algorithm is also iterative, implying there can be multiple Gaussian eliminations. In this case, $\underline{L}_p^{(t)}$ will be mapped to $\underline{L}_a^{(t)}$, triggering another round of sorting and Gaussian elimination. The decoding at level t will terminate either when the message is decoded or when the maximum number of ABP iteration (denoted as N_{ABP}) is reached. In this work, each level RS decoding outcome is validated by the maximum likelihood criterion [11].

IV. Design of MRS Codes

With the multilevel coding scheme, the transmission of a symbol x_j over the channel can be decomposed into parallel transmission of m individual bits $(c_j^{(0)}, c_j^{(1)}, \ldots, c_j^{(m-1)})$ over m equivalent channels $(0, 1, \ldots, m-1)$. We can analyze the capacity of each equivalent channel and design the code rate of each level accordingly [12]. With a competent decoding algorithm, this design approach can achieve error-free transmission over each equivalent channel. As a result, the overall IMSD performance can be enhanced.

Let $\mathcal{C}^{(t)}$ denote the capacity of equivalent channel t. For an MRS coded transmission, the overall capacity $\mathcal{C}$ is

$$\mathcal{C} = \sum_{t=0}^{m-1} \mathcal{C}^{(t)}. \tag{23}$$

Given a competent decoding mechanism, when $\mathcal{R}^{(t)} = \mathcal{C}^{(t)}$, the MRS code can approach capacity $\mathcal{C}$. Let $X^{(t)}$ and Y denote the transmitted and received variables at equivalent channel t, respectively. The capacity of equivalent channel t can be determined as the conditional mutual information $I(Y; X^{(t)} \mid X^{(0)} \cdots X^{(t-1)})$ [1]. Based on the chain rule for mutual information [13]

$$I(Y; X^{(0)} \cdots X^{(m-1)}) = \sum_{t=0}^{m-1} I(Y; X^{(t)} \mid X^{(0)} \cdots X^{(t-1)}), \tag{24}$$

the capacity of equivalent channel t can be determined by

$$\begin{aligned}\mathcal{C}^{(t)} = &\, I(Y; X^{(t)} \cdots X^{(m-1)} \mid X^{(0)} \cdots X^{(t-1)}) \\ &- I(Y; X^{(t+1)} \cdots X^{(m-1)} \mid X^{(0)} \cdots X^{(t)}).\end{aligned} \tag{25}$$

Let $\mathcal{C}(\mathcal{B})$ denote the channel capacity that is associated with a modulated symbol set $\mathcal{B}$. Given an M-ary constellation and the knowledge of bits $\theta_0, \ldots, \theta_{t-1}$, we can define the symbol set

$$\mathcal{S}(\theta_0 \cdots \theta_{t-1}) = \{s_i \mid [\mathcal{M}^{-1}(s_i)]_\tau = \theta_\tau, \forall\, 0 \le \tau \le t-1\}. \tag{26}$$

Since there are 2^t permutations of $[\theta_0, \ldots, \theta_{t-1}]$ and their associated symbol sets $\mathcal{S}(\theta_0 \cdots \theta_{t-1})$,

$$\begin{aligned}&I(Y; X^{(t)} \cdots X^{(m-1)} \mid X^{(0)} \cdots X^{(t-1)}) \\ &= \frac{1}{2^t} \sum_{\mathcal{S}(\theta_0 \cdots \theta_{t-1})} \mathcal{C}(\mathcal{S}(\theta_0 \cdots \theta_{t-1})).\end{aligned} \tag{27}$$

Similarly,

$$\begin{aligned}&I(Y; X^{(t+1)} \cdots X^{(m-1)} \mid X^{(0)} \cdots X^{(t)}) \\ &= \frac{1}{2^{t+1}} \sum_{\mathcal{S}(\theta_0 \cdots \theta_t)} \mathcal{C}(\mathcal{S}(\theta_0 \cdots \theta_t)).\end{aligned} \tag{28}$$

Based on (25), the capacity of equivalent channel t is

$$\begin{aligned}\mathcal{C}^{(t)} = &\frac{1}{2^t} \sum_{\mathcal{S}(\theta_0 \cdots \theta_{t-1})} \mathcal{C}(\mathcal{S}(\theta_0 \cdots \theta_{t-1})) \\ &- \frac{1}{2^{t+1}} \sum_{\mathcal{S}(\theta_0 \cdots \theta_t)} \mathcal{C}(\mathcal{S}(\theta_0 \cdots \theta_t)).\end{aligned} \tag{29}$$

[1]It is assumed that $I(Y; X^{(t)} \mid X^{(0)} \cdots X^{(t-1)})$ is maximized by an appropriate distribution of $X^{(t)}$.

In particular, over the additive white Gaussian noise (AWGN) channel, capacity $\mathcal{C}(\mathcal{B})$ can be determined by

$$\mathcal{C}(\mathcal{B}) = \frac{1}{|\mathcal{B}|} \sum_{s_i \in \mathcal{B}} \int_{y_j} P(y_j \mid s_i) \log_2 \left(\frac{P(y_j \mid s_i)}{\frac{1}{|\mathcal{B}|} \sum_{s_\xi \in \mathcal{B}} P(y_j \mid s_\xi)} \right) dy_j, \tag{30}$$

in which it is assumed that each symbol of $\mathcal{B}$ is equiprobable for transmission.

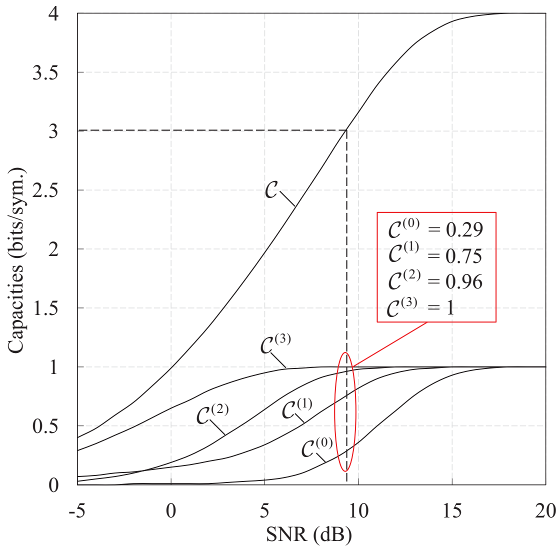

Fig. 2. Equivalent channel capacities of the SP 16-QAM constellation.

Therefore, we can design the level t RS code by adjusting its rate $\mathcal{R}^{(t)}$ as closed to $\mathcal{C}^{(t)}$ as possible. Fig. 2 shows the equivalent channel capacities of the set partitioning (SP) 16-QAM constellation over the AWGN channel. The SP constellation is designed by progressively enlarging the intraset minimum Euclidean distance of the partitioned subsets. It can therefore maximize the multistage decoding performance for MRS codes. It can be seen that to achieve an overall capacity of 3 bits/sym., the equivalent channel capacities are $\mathcal{C}^{(0)} = 0.29$ bits/sym., $\mathcal{C}^{(1)} = 0.75$ bits/sym., $\mathcal{C}^{(2)} = 0.96$ bits/sym. and $\mathcal{C}^{(3)} = 1$ bit/sym., respectively. This indicates when using the SP 16-QAM, the HeMRS codes should be employed to achieve the channel capacity.

V. Performance Analysis

We now show the bit error rate (BER) performance of MRS codes using the IMSD algorithm. The simulation results are obtained over the AWGN channel using the SP 16-QAM. In our simulations, we let $N_{\text{ABP}} = 3$ and $\eta = 0.12$. Three BP iterations are performed based on each adapted parity-check matrix $\mathcal{H}'_t$.

Fig. 3 shows the IMSD performance of the HoMRS code which employs the (63, 47) RS codes as the component codes. The proposed IMSD algorithm is compared with the conventional IMD algorithm in which the RS codes are decoded by the BM algorithm. It iterates hard decisions between different levels. We can see that the proposed IMSD algorithm

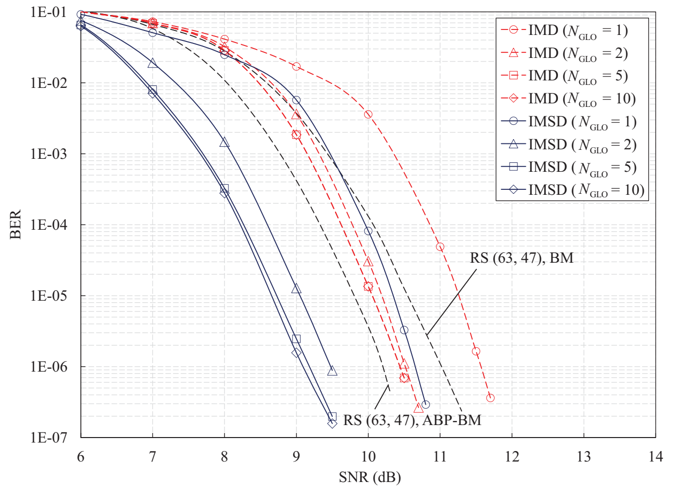

Fig. 3. IMSD performance of the HoMRS code using SP 16-QAM.

outperforms the hard-decision IMD algorithm. The decoding performance can be enhanced by increasing the maximum iteration number ($N_{\rm GLO}$). In contrast, the conventional IMD algorithm yields no performance improvement by increasing its iteration number beyond five. Fig. 3 also compares the performance of the HoMRS code with a single (63, 47) RS code that employs the same modulation. It shows that without iteration, the HoMRS coded transmission performs worse than a signal RS code. This is because in the MRS coding scheme, m bits of m individual codes are bundled for transmission. If the channel introduces an error, the error may reside at all levels, degrading the decoding performance. However, when the decoding starts to iterate, performance advantage of multilevel coding becomes remarkable.

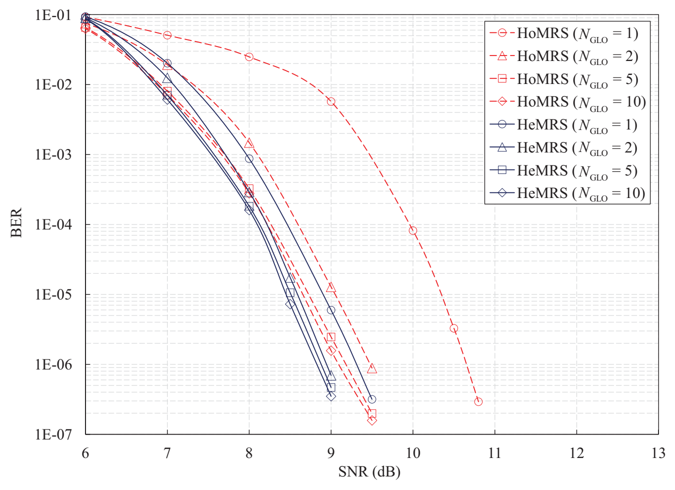

Fig. 4. Performance comparison between the HeMRS and the HoMRS codes.

Fig. 4 compares the IMSD performance of the HeMRS and the HoMRS codes when employing the SP 16-QAM. The HeMRS code is designed based on the equivalent channel capacities shown in Fig. 2. To achieve an overall capacity of 3 bits/sym., component codes at levels 0, 1 and 2 are the (63, 17), the (63, 47) and the (63, 59) RS codes, respectively, while level 3 is uncoded. The HoMRS code employs the (63, 47) RS codes as its component codes. Fig. 4 shows that the HeMRS code outperforms its homogeneous counterpart. Fig. 2 shows that to realize a capacity of 3 bits/sym., a signal-to-noise ratio (SNR) of 9.32dB is needed for the SP 16-QAM. Fig. 4 shows the HeMRS code can approach this limit. This demonstrates the effect of our proposed MRS code design.

VI. Conclusions

This paper has proposed an IMSD for MRS codes, achieving both high decoding performance and transmission spectrum efficiency. The RS decoding is realized by the ABP-BM algorithm, delivering either the extrinsic or the deterministic probabilities for the coded bits. Over a multistage decoding mechanism, the earlier decoding outputs facilitate the later decoding events. We have further proposed the design of MRS codes based on analyzing the equivalent channel capacity of each coded level, leading to the HeMRS codes. Our simulation results have verified the performance advantage of the IMSD algorithm and the designed MRS codes can approach the channel capacity.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (NSFC) with project ID 61671486.

References

[1] H. Imai and S. Hirakawa, “A new multilevel coding method using error-correcting codes,” *IEEE Trans. Inform. Theory*, vol. 23, no. 3, pp. 371–377, May 1977.

[2] P. A. Martin and D. P. Taylor, “On multilevel codes and iterative multistage decoding,” *IEEE Trans. Commun.*, vol. 49, no. 11, pp. 1916–1925, Nov. 2001.

[3] M. Isaka and H. Imai, “On the iterative decoding of multilevel codes,” *IEEE J. Sel. Areas Commun.*, vol. 19, no. 5, pp. 935–943, May 2001.

[4] Y. Wang and A. G. Burr, “Code design for iterative decoding of multilevel codes,” *IEEE Trans. Commun.*, vol. 63, no. 7, pp. 2404–2419, Jul. 2015.

[5] S. Y. Chung and H. Lou, “Multilevel RS/convolutional concatenated coded QAM for hybrid IBOC-AM broadcasting,” *IEEE Trans. Broadcast.*, vol. 46, no. 1, pp. 49–59, Mar. 2000.

[6] E. M. Husni and P. Sweeney, “Robust Reed Solomon coded MPSK modulation,” *ITB J. Inform. Commun. Technol.*, vol. 4, no. 2, pp. 95–113, 2010.

[7] A. Vardy and Y. Be’Ery, “Bit-level soft-decision decoding of Reed-Solomon codes,” *IEEE Trans. Commun.*, vol. 39, no. 3, pp. 440–444, Mar. 1991.

[8] V. Ponnampalam and B. Vucetic, “Soft decision decoding of Reed-Solomon codes,” *IEEE Trans. Commun.*, vol. 50, no. 11, pp. 1758–1768, Nov. 2002.

[9] J. Jiang and K. R. Narayanan, “Iterative soft-input soft-output decoding of Reed-Solomon codes by adapting the parity-check matrix,” *IEEE Trans. Inform. Theory*, vol. 52, no. 8, pp. 3746–3756, Aug. 2006.

[10] J. L. Massey, “Shift-register synthesis and BCH decoding,” *IEEE Trans. Inform. Theory*, vol. 15, no. 1, pp. 122–127, Jan. 1969.

[11] T. Kaneko, T. Nishijima, H. Inazumi, and S. Hirasawa, “An efficient maximum-likelihood-decoding algorithm for linear block codes with algebraic decoder,” *IEEE Trans. Inform. Theory*, vol. 40, no. 2, pp. 320–327, Mar. 1994.

[12] U. Wachsmann, R. F. H. Fischer, and J. B. Huber, “Multilevel codes: theoretical concepts and practical design rules,” *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1361–1391, Jul. 1999.

[13] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley, 2003.