Error Probability Analysis of M-QAM on Rayleigh Fading Channels with Impulsive Noise

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***Abstract*—In conventional communication systems, the noise at the receiver is usually assumed to be Gaussian. However, this assumption is not always valid if the signal is affected by impulsive noise. This paper analyzes the theoretical bit error probability (BEP) of M-ary quadrature amplitude modulation (M-QAM) ($M \geq 4$) on Rayleigh fading channels with impulsive noise that has an alpha stable distribution. The derived theoretical BEP can be used to evaluate the performance of wireless communication system with impulsive noise and also provides a benchmark for determining coding gains. Furthermore, two closed-form approximations of the theoretical BEP are derived that provide good approximations of the actual BEP.**

***Index terms*— bit error probability, symmetric alpha stable distribution, Rayleigh fading**

I. Introduction

The noise at the receiver in a practical wireless communication system is not always Gaussian due to other sources of noise, such as impulsive noise. Many sources can generate natural or man-made impulsive interference, such as switching transients in power lines, underwater acoustic noise and multiple access interference (MAI) [1, 2]. This interference can be characterized by a the occurrence of large noise samples, which results in a heavy-tailed distribution. In this case, the Gaussian noise model is not correct and many distributions such as Bernoulli-Gaussian model, Middleton Class A, B and Symmetric Alpha Stable (SαS) distributions are more suitable to model these impulsive channels [3–5]. Recently, SαS noise was employed as an accurate model of multiple access interference (MAI) in a wireless *ad hoc* network and as near-field interference in wireless transceivers [2, 6]. Additionally, for underwater acoustic channel modeling, diversity combining methods and space time coding over Rayleigh fading channels with SαS noise were investigated in [7–9].

Due to the lack of closed-form probability density function (pdf), most literature proposes sub-optimal receivers combined with different coding schemes for SαS channels [10–12]. Hence, an analytic expression of BEP is needed to evaluate the performance of a wireless system with impulsive noise. Any complex SαS variable can be classified as a sub-Gaussian variable where its real and imaginary components are dependent. However, if the passband sampling frequency f_s is four times the carrier frequency f_c, they are known to be independent [13] and this type of channel has been used in the literature to model impulsive noise [7, 14, 15]. This also provides a way to analyze the performance of M-ary modulation schemes.

In this paper, we derive the BEP of M-QAM for flat Rayleigh fading channels with SαS distributed noise and simplify it by using an alternate expression of the cumulative distribution function (CDF) of standard SαS distributions. In addition, two closed-form approximations of BEP are proposed which greatly reduce the computational cost without sacrificing much accuracy. One method is the Bi-parameter Cauchy-Gaussian mixture (BCGM) model which approximates the exact pdf in closed-form [16]. Another method employs an asymptotic property of SαS distributions, which is a good estimate of the high signal-to-noise ratio (SNR) region of the BEP.

This paper is organized as follows: Section II describes the system model and introduces the geometric signal-to-noise ratio ($\mathrm{SNR_G}$) for SαS noise. Section III derives the BEP of M-QAM with baseband SαS noise and this is extended to Rayleigh fading channels with SαS noise. Moreover, two approximations of exact BEP and their simplification are derived in Section IV. In Section V, theoretical and simulation results are presented and we conclude the paper in Section VI.

II. System model

We begin by defining a point-to-point system with a coherent receiver and the n-th received signal sample $y(n)$ is expressed as

$$y(n) = hx(n) + z(n), \tag{1}$$

where $x(n)$ is modulated signal with M-QAM modulation and h is the complex Gaussian fading coefficient with zero mean and unit variance and the envelope a has a Rayleigh pdf. $z(n)$ is complex noise where the real part $z_R(n)$ and imaginary part $z_I(n)$ are identically independent distributed (i.i.d.) and follow the symmetric alpha-stable (SαS) distribution. The characteristic function of alpha-stable distributions is

$$\varphi(t) = \exp\left\{j\delta t - \mid \gamma t \mid^{\alpha} (1 - j\beta \mathrm{sign}(t)\omega(t,\alpha))\right\}. \tag{2}$$

where

$$\omega(t,\alpha) = \begin{cases} \tan(\pi\alpha/2), & \alpha \neq 1 \\ -2/\pi \log|t|. & \alpha = 1 \end{cases}$$

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The alpha-stable distribution $S(\alpha,\beta,\gamma,\delta)$ in (2) has four parameters, α, β, γ and δ. 1) The characteristic exponent α, has a range $(0,2]$ and determines the tail heaviness; 2) the skewness is denoted by β; 3) the dispersion is denoted by γ^α, which determines the spread of the pdf; 4) the location parameter is denoted as δ [5]. The alpha-stable distribution is called symmetric if β and δ are 0. Hence the pdf of a SαS distribution is

$$f_\alpha(x;\gamma)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-\gamma^\alpha|t|^\alpha)e^{-jtx}dt. \tag{3}$$

There are two special cases in the SαS family which have closed form expressions. When $\alpha=1$, the noise is Cauchy. When $\alpha=2$, the noise is Gaussian and the variance σ^2 is only defined in this case with $\sigma^2=2\gamma^2$.

To generate independent complex SαS noise, we note that any SαS random variable $w\sim S(\alpha,0,\gamma,0)$ can be written as a compound Gaussian $w=\sqrt{B}G$, where B and G are independent, with $B\sim S(\alpha/2,1,[\cos(\pi\alpha/4)]^{2/\alpha},0)$ and G is a Gaussian random variable with zero mean and variance σ^2 [17]. In our case, since $z_R(n)$ and $z_I(n)$ are independent, $z(n)$ can be described as

$$z(n)=\sqrt{B_1}G_1+j\sqrt{B_2}G_2, \tag{4}$$

where B_1 and B_2 are i.i.d. and are distributed like B. Similarly, G_1 and G_2 are i.i.d. Gaussian random variables which follow $\mathcal{N}(0,\sigma^2)$.

The conventional signal-to-noise ratio (SNR) is not defined for SαS noise since the second order moment of SαS process is infinite. Hence, we use the geometric SNR ($\mathrm{SNR_G}$) instead [18]. The geometric power S_0 is defined as

$$S_0=\frac{(C_g)^{1/\alpha}\gamma}{C_g}, \tag{5}$$

where C_g is the exponential of the Euler constant and $C_g\approx 1.78$. The $\mathrm{SNR_G}$ is defined as

$$\mathrm{SNR_G}=\frac{1}{2\mathrm{C_g}}\left(\frac{\mathrm{A}}{\mathrm{S_0}}\right)^2, \tag{6}$$

where A^2 is the transmitted energy of the modulated signal and the constant $\frac{1}{2C_g}$ ensures $\mathrm{SNR_G}$ remains valid when the noise is Gaussian (when $\alpha=2$). Hence the $\frac{E_b}{N_0}$ for M-QAM modulation is given as

$$\frac{E_b}{N_0}=\frac{A^2}{4\log_2(M)C_g^{(\frac{2}{\alpha}-1)}\gamma^2}, \tag{7}$$

where $A^2=E_s$. When $M=2$ and $A=1$, (7) becomes the $\frac{E_b}{N_0}$ for binary phase-shift keying (BPSK).

III. Analytic BEP of M-QAM on Rayleigh Fading Channels with SαS Noise

We start with the derivation of the BEP of BPSK with SαS noise. First the tail probability $Q_\alpha(x)$ for SαS distributions is defined, which is similar to the well known Q-function.

$$Q_\alpha(x)=\int_x^{\infty}f_\alpha^s(t)dt, \tag{8}$$

where $f_\alpha^s(x)$ is the standard SαS distribution by setting $\gamma=1$ and is given as

$$f_\alpha^s(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(-|t|^\alpha)e^{-jtx}dt. \tag{9}$$

We can derive the BEP for SαS channels from Cauchy noise (i.e. $\alpha=1$), since the Cauchy distribution is the only heavy tailed distribution in the SαS distributions that has a closed form pdf. When $\alpha=1$, the BEP of BPSK with Cauchy noise is denoted as

$$\begin{aligned}P_{b,\alpha}^{\mathrm{B}}&=P(x=+1)P(e|x=+1)+P(x=-1)P(e|x=-1)\\&=\int_{\frac{1}{\gamma}}^{\infty}\frac{1}{\pi}\frac{1}{x^2+1}dx,\end{aligned} \tag{10}$$

where $P(e|x=-1)$ is the conditional probability that an error occurs and $P(x=+1)=P(x=-1)=\frac{1}{2}$. The Cauchy distribution of (10) is a standard pdf, hence $P_{b,\alpha}^{\mathrm{B}}$ can also be expressed by $Q_\alpha(x)$ as

$$P_{b,\alpha}^{\mathrm{B}}=Q_\alpha\left(\frac{1}{\gamma}\right)=Q_\alpha\left(\sqrt{4C_g^{(\frac{2}{\alpha}-1)}\frac{E_b}{N_0}}\right). \tag{11}$$

Since $\mathrm{SNR_G}$ is defined for all α's, $P_{b,\alpha}^{\mathrm{B}}$ in (11) can be seen as a general expression for the BEP of a uncoded system with SαS noise. A special case of the SαS family is the Gaussian distribution ($\alpha=2$), which is the only $S\alpha S$ distribution that has exponential tails. Notice that the variance of the normal distribution when expressed as an SαS pdf is equal to two, not one. Hence $f_\alpha^s(t)=\frac{1}{2\sqrt{\pi}}\exp(-\frac{t^2}{4})$ is the standard SαS distribution when $\alpha=2$. Then the BEP of BPSK on the AWGN channel can be expressed in terms of the Q_α-function as

$$P_b^{\mathrm{Gauss}}=Q\left(\sqrt{\frac{2E_b}{N_0}}\right)=Q_{\alpha=2}\left(2\sqrt{\frac{E_b}{N_0}}\right). \tag{12}$$

When $\alpha=2$, (11) reduces to (12) which implies (11) is a general expression for all α's. The mapping between $Q(x)$ and $Q_\alpha(x)$ is

$$Q(x)\to Q_\alpha\left(\sqrt{2C_g^{(\frac{2}{\alpha}-1)}}x\right). \tag{13}$$

We can find the BEP of SαS channels according to (13). For AWGN channels, according to [19], the closed-form BEP of M-QAM modulation is given as

$$\begin{aligned}P_b^{\mathrm{M}}=&\frac{2}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1}\\&\left\{f(k,i)Q\left((2i+1)\sqrt{\frac{3\log_2 M\cdot\Omega}{M-1}}\right)\right\},\end{aligned} \tag{14}$$

where $\Omega=\frac{E_b}{N_0}$ and $\lfloor x\rfloor$ represents the largest integer that is not greater than x. $f(k,i)$ is denoted as

$$f(k,i)=(-1)^{\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}\right\rfloor}\left(2^{k-1}-\left\lfloor\frac{i\cdot 2^{k-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right). \tag{15}$$

The relationship between $Q(x)$ and $Q_\alpha(x)$ is given in (13), hence the theoretical BEP of M-QAM over SαS noise is given as

$$P_{b,\alpha}^{\mathrm{M}} = \frac{2}{\sqrt{M}\log_2\sqrt{M}} \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \left\{ f(k,i) Q_\alpha \left((2i+1)\sqrt{\frac{6C_g^{(\frac{2}{\alpha}-1)}\log_2 M\cdot\Omega}{M-1}} \right) \right\} \tag{16}$$

Now we consider flat Rayleigh fading channels. For a fixed attenuation a, the error rate of M-QAM in (16) is viewed as a conditional error probability $P_{b|a,\alpha}^{\mathrm{M}}$ where Ω is replaced by $\omega = a^2\frac{E_b}{N_0}$. To obtain the error probability when a is random, we must average $P_{b|a,\alpha}^{\mathrm{M}}$, over the pdf of ω. The general expression of the exact BEP on the Rayleigh fading channels with SαS noise for M-QAM is given as

$$P_{b,\alpha}^{\mathrm{M,Ray}} = \int_0^\infty P_{b|a,\alpha}^{\mathrm{M}}(\omega)p(\omega)d\omega \tag{17}$$

where the pdf of ω is $p(\omega) = \frac{1}{\Omega}\exp(-\omega/\Omega)$. We note that the complexity of calculating the exact BEP in (17) is very high. Since the pdf of SαS in (9) is an integral, we will need a triple integral to calculate (17).

We present a method to simplify (17) to a double integral by reducing the complexity of calculating $Q_\alpha(x)$. We observe that $Q_\alpha(x) = 1 - F_\alpha(x)$, where $F_\alpha(x)$ is the CDF of standard SαS distribution. Hence, we can use an alternative representation of $F_\alpha(x)$ which only contains one integral [20]. For $x > 0$:
(a) When $\alpha > 1$,

$$F_\alpha(x) = 1 - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-x^{\frac{\alpha}{\alpha-1}}V(\theta;\alpha)\right)d\theta, \tag{18}$$

where

$$V(\theta;\alpha) = \left(\frac{\cos\theta}{\sin\alpha\theta}\right)^{\frac{\alpha}{\alpha-1}} \frac{\cos(\alpha-1)\theta}{\cos\theta}. \tag{19}$$

(b) When $\alpha = 1$,

$$F_\alpha(x) = \frac{1}{2} + \frac{1}{\pi}\arctan(x). \tag{20}$$

(c) When $\alpha < 1$,

$$F_\alpha(x) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-x^{\frac{\alpha}{\alpha-1}}V(\theta;\alpha)\right)d\theta. \tag{21}$$

In this way, the computational cost of $Q_\alpha(x)$ ($\alpha \neq 1$) is reduced from a double integral to a single integral. To further decrease the complexity, we will next propose two methods to approximate the BEP with very simple expressions and with only a small loss in accuracy.

IV. Two Approximations of BEP of M-QAM on Rayleigh fading channels with SαS noise

A. BEP approximation from BCGM model

The BCGM model was proposed to approximate SαS distributions ($1 \leq \alpha \leq 2$) by mixing a Gaussian distribution ($\alpha = 2$) and a Cauchy distribution ($\alpha = 1$) with only two necessary parameters, ϵ and γ. Hence, we employ the BCGM distribution to approximate α-stable pdf. The BCGM pdf is given as

$$f_{\mathrm{CG}}(x) = (1-\epsilon)\frac{1}{2\sqrt{\pi}\gamma}\exp\left(-\frac{x^2}{4\gamma^2}\right) + \epsilon\frac{\gamma}{\pi(x^2+\gamma^2)}, \tag{22}$$

where ϵ is the mixture ratio and it achieves a near optimum value when

$$\varepsilon = \frac{2\Gamma(-p/\alpha) - \alpha\Gamma(-p/2)}{2\alpha\Gamma(-p) - \alpha\Gamma(-p/2)}. \tag{23}$$

The gamma function is $\Gamma(x) = \int_0^\infty e^{-t}t^{x-1}dt$ and $p < \alpha$. Then We can define the standard BCGM distribution as

$$f_{\mathrm{CG}}^s(x) = (1-\varepsilon)\frac{1}{2\sqrt{\pi}}\exp\left(-\frac{x^2}{4}\right) + \varepsilon\frac{1}{\pi(x^2+1)} \tag{24}$$

The BEP of an SαS channel can be approximated by $f_{\mathrm{CG}}^s(x)$ as

$$P_{b,\alpha}^{\mathrm{M,BCGM}} = \frac{2}{\sqrt{M}\log_2\sqrt{M}} \sum_{k=1}^{\log_2\sqrt{M}} \sum_{i=0}^{(1-2^{-k})\sqrt{M}-1} \left\{ f(k,i)\int_{\sqrt{g(i)\Omega}}^\infty f_{\mathrm{CG}}^s(x)dx \right\} \tag{25}$$

where

$$g(i) = \frac{6C_g^{(\frac{2}{\alpha}-1)}\log_2 M}{M-1}(2i+1)^2 \tag{26}$$

The exact BEP on Rayleigh fading channels with SαS noise is now approximated by using the BCGM model as

$$P_{b,\alpha}^{\mathrm{M,Ray}} \approx \frac{1}{\Omega}\int_0^\infty P_{b|a,\alpha}^{\mathrm{M,BCGM}}(\omega)\exp(-\omega/\Omega)d\omega. \tag{27}$$

It is observed that (27) contains a double integral. After some manipulation, a closed-form expression of BEP can be obtained and is given in (28). This greatly reduces the computational cost compared with the exact BEP. When $\epsilon = 0$, (28) is the exact BEP for Rayleigh fading with Gaussian noise, and when $\epsilon = 1$, (28) becomes the exact BEP of Rayleigh fading with Cauchy noise.

B. Asymptotic performance of a Rayleigh fading channel with SαS noise

Another approximation can be obtained by using the heavy tailed property of SαS distributions. According to [5], for a α-stable random variable X with dispersion γ^α, we have

$$\lim_{x\to\infty} P(X > x) = \frac{\gamma^\alpha C_\alpha}{x^\alpha}, \tag{29}$$

where

$$C_\alpha = \frac{1}{\pi}\Gamma(\alpha)\sin\left(\frac{\pi\alpha}{2}\right). \tag{30}$$

$$P_{b,\alpha}^{\mathrm{M,Ray}} \approx \frac{1}{\Omega}\int_0^\infty P_{b|a,\alpha}^{\mathrm{M,BCGM}}(\omega)\exp(-\omega/\Omega)d\omega$$
$$= \frac{2}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1}$$
$$\left\{f(k,i)\left[\frac{1-\epsilon}{2}\left(1-\sqrt{\frac{g(i)\Omega}{4+g(i)\Omega}}\right)+\frac{\epsilon}{2}\left(1-\exp\left(\frac{1}{g(i)\Omega}\right)\mathrm{erfc}\left(\sqrt{\frac{1}{g(i)\Omega}}\right)\right)\right]\right\} \tag{28}$$

Hence the asymptotic tail probability $Q_\alpha(x)$ is given as

$$\lim_{x\to\infty} Q_\alpha(x) = \frac{C_\alpha}{x^\alpha}, \tag{31}$$

By substituting (31) into (16), we obtain the asymptotic BEP for M-QAM on SαS channels

$$P_{b,\alpha}^{\mathrm{M,asy}} = \frac{2C_\alpha}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1}$$
$$\left\{f(k,i)\left(g(i)\Omega\right)^{-\frac{\alpha}{2}}\right\}. \tag{32}$$

After some transformations, the asymptotic BEP on Rayleigh fading channels with SαS noise for M-QAM is given as

$$P_{b,\alpha}^{\mathrm{M,Ray}} \to \frac{1}{\Omega}\int_0^\infty P_{b|a,\alpha}^{\mathrm{M,asy}}(\omega)\exp(-\omega/\Omega)d\omega$$
$$= \frac{2C_\alpha}{\sqrt{M}\log_2\sqrt{M}}\sum_{k=1}^{\log_2\sqrt{M}}\sum_{i=0}^{(1-2^{-k})\sqrt{M}-1}$$
$$\left\{f(k,i)\left(g(i)\Omega\right)^{-\frac{\alpha}{2}}\Gamma\left(1-\frac{\alpha}{2}\right)\right\}. \tag{33}$$

The resulting expression of the asymptotic BEP is also very simple, containing only a gamma function. We note that for slightly impulsive Rayleigh fading channels (i.e. $\alpha = 1.8$), only the first two terms ($i = 0, 1$) of (17), (28) and (33) are required to give a good estimate of the BEP. However, when the channel becomes more impulsive (i.e. $\alpha = 1$), more terms ($i = 0, 1, 2...$) are considered since the tail probability decays slowly as SNR increases.

V. Simulation Results

Simulation results and numerical results for the closed-form expressions of the exact and approximated BEP of M-QAM on Rayleigh fading channels with SαS noise are presented in this section.

As shown in Fig. 1, when $\alpha = 1.9$ or the channel is slightly impulsive, our analytic BEPs match the simulation results for different orders of M-QAM. The BCGM model approximates the exact BEP very accurately at the high error-rate region. For example, when $\alpha = 1.9$, the BCGM BEP curve for each M-QAM scheme matches the theoretical BEP when BEP $> 10^{-2}$. When SNR increases, this estimation becomes less accurate. In contrast, the asymptotic BEPs closely approximate the low error-rate region of the exact BEPs, but the approximation is less accurate in the high error-rate region. As seen in Fig. 1, for each M-QAM scheme, the asymptotic BEP matches the theoretical BEP very closely when the BEP is equal to or less than 10^{-2}.

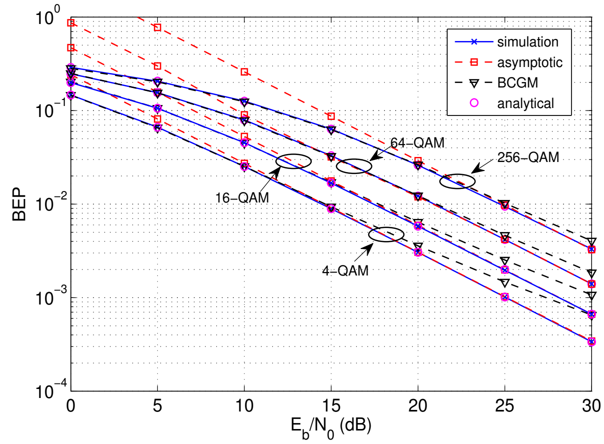

Fig. 1. BEP of M-QAM on Rayleigh fading channels with SαS noise when $\alpha = 1.9$.

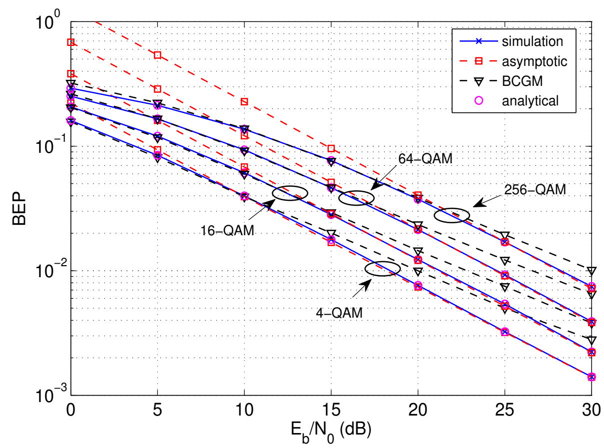

Fig. 2. BEP of M-QAM on Rayleigh fading channels with SαS noise when $\alpha = 1.5$.

When the channel becomes more impulsive with $\alpha = 1.5$, our theoretical BEPs are still closely approximate simulated bit-error rate curves in Fig. 2. Similar to the case of $\alpha = 1.9$, the asymptotic BEP accurately approximates the low error-

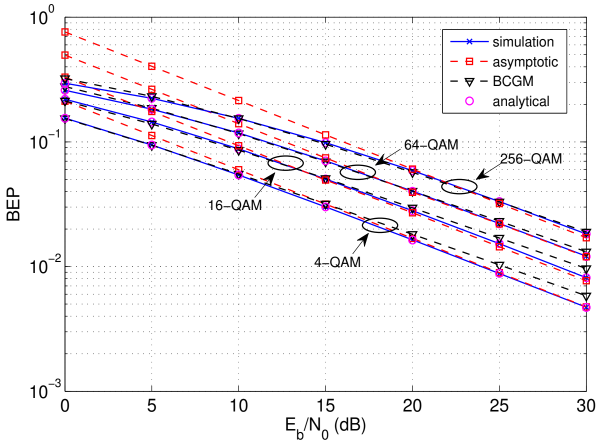

Fig. 3. BEP of M-QAM on Rayleigh fading channels with SαS noise when $\alpha = 1.1$.

rate region for each M-QAM scheme. A different result is the BCGM modeled BEP, which is less accurate compared with $\alpha = 1.9$. Fig. 2 shows that the BCGM model closely approximates the exact BEP when BEP $> 3 \times 10^{-2}$.

When the channel is highly impulsive with $\alpha = 1.1$, as shown in Fig. 3, our BEP approximations are still verified by simulation results. In this case, the BCGM model is more accurate compared with the case when $\alpha = 1.5$. This is because the BCGM model better approximates the SαS pdf when α approaches 2 or 1. The numerical results of the asymptotic BEP accurately capture the low error-rate region performance as before, which implies that the asymptotic BEP is robust for all levels of impulsiveness.

VI. Conclusion

In this paper, we have derived the exact BEP of M-QAM on Rayleigh fading channels with SαS noise to evaluate the performance of a wireless communication system with impulsive noise. We have also presented two approximations of the exact BEP, based on the BCGM model and an asymptotic approximation. The BCGM model matched the high error-rate region of the exact BEP curve and increased in accuracy when the channel was lightly or heavily impulsive. However, this method became less accurate when the channel was moderately impulsive ($\alpha = 1.5$) because it does not model the pdf as accurately. The asymptotic BEP consistently provided a good approximation for the BEP for all levels of impulsiveness in the low error-rate region. Most importantly, these two approximations have closed-from expressions, which greatly reduces the computation complexity.

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