Design of BICM-ID for Two-Way Relay Channels

Li Chen †, Yushan Yakufu †, Xiaojun Yuan ‡ and Zichao Sun †
† School of Information Science and Technology, Sun Yat-sen University, Guangzhou, China
‡ School of Information Science and Technology, ShanghaiTech University, Shanghai, China
Emails: chenli55@mail.sysu.edu.cn, yakufu@mail2.sysu.edu.cn, yuanxj@shanghaitech.edu.cn, sunzch@mail2.sysu.edu.cn

***Abstract*—This paper investigates the iterative decoding for spectrally efficient two-way relay (TWR) communications in which two end nodes exchange information through a relay (R). With the use of physical-layer network coding (PNC), information exchange throughput can be significantly improved as only two orthogonal time slots (TS) are required to realize both the multiple access (MAC) and the broadcast (BC) transmissions. During the MAC stage, transmitted symbols from the end nodes are superimposed at R which decodes the exclusive-or (XOR) information of the two end nodes. However, with a high order modulation, symbol superposition will cause demapping ambiguity. This paper proposes a TWR communication system that employs the bit-interleaved coded modulation (BICM). The iterative decoding, namely, the BICM-ID, is introduced to dissolve the demapping ambiguity. With iterative decoding, the transmitted symbol pair probabilities can be better estimated so that the demapper can provide better coded bit information for the decoder. The proposed work generalizes the iterative demapping-decoding approach for TWR channels. Our simulation results demonstrate significant iterative decoding gains can be achieved with a tendency of approaching the XOR-decoding bound.**

***Index Terms*—BICM, iterative decoding, physical-layer network coding, spectrum efficiency, two-way relay**

I. Introduction

Network coding (NC) [1] is a celebrated concept for improving the information throughput in a communication network. Among its many applications, two end nodes without direct link exchange information through a relay (R) is a scenario in which NC plays an important role. As shown in Fig. 1(a), nodes A and B wish to exchange binary message vectors $\overline{u}_{\mathrm{A}}$ and $\overline{u}_{\mathrm{B}}$, respectively. During the multiple access (MAC) stage, A and B transmit their message vectors through two orthogonal time slots (TSs). If they can be correctly decoded by R, R generates an exclusive-or (XOR) message vector $\overline{u}_{\mathrm{A}} \oplus \overline{u}_{\mathrm{B}}$ and broadcasts it to A and B during the broadcast (BC) stage. With the knowledge of its own message, an end node can recover the intended message if it can correctly decode $\overline{u}_{\mathrm{A}} \oplus \overline{u}_{\mathrm{B}}$. Physical-layer NC (PNC) [2]–[4] was introduced by further relaxing the time orthogonality of the MAC stage. R decodes $\overline{u}_{\mathrm{A}} \oplus \overline{u}_{\mathrm{B}}$ with the observation of the superimposed signals from A and B. As shown in Fig. 1(b), A and B exchange information using only two TSs. This PNC assisted information exchange is also called the two-way relay (TWR) communications and the channel experienced by R during the MAC stage is called the TWR channel. It has been shown that TWR communications can significantly improve the information throughput [4]. The capacity region of the end nodes in the TWR Gaussian channel has been characterized by [5]. In approaching the capacity, lattice code has been considered in [5]–[7] with an attempt to harness the MAC transmission interference. However, lattice code is yet to be a practical channel code due to its implementation complexity. In utilizing the conventional channel codes to decode the XOR message, repeat-accumulate (RA) codes have been considered with redesigning the decoding algorithm by [8]. Convolutional coded TWR communications was considered and a reduced complexity decoding algorithm was proposed in [9]. The designs of irregular RA (IRA) codes and low-density parity-check (LDPC) codes for TWR Gaussian channels have been investigated in [10] and [11], respectively. However, the above mentioned works employed BPSK modulation limiting the spectrum efficiency of TWR communications. The lack of work in using high order modulation schemes is due to the fact that demapping ambiguity usually arises as one superimposed symbol may correspond to multiple XOR coded bit permutations. So far, only a few works [12] [13] have considered using high order modulation schemes. They attempted to avoid the ambiguity problem by redesigning the modulation scheme, such that the superimposed symbol and the XOR coded bit permutation exhibit a one-to-one map. However, only uncoded scenarios were considered in [12] [13], which limits the scope of their potential applications. Recently, the iterative noncoherent receiver design for TWR communications was considered by [14], in which the end nodes employ the multi-tone frequency shift keying (FSK). It realizes spectrally efficient TWR communications, and fortunately FSK does not cause the demapping ambiguity at R. However, it is understood that FSK modulation will incur a more complex receiver structure than other energy efficient modulation schemes, e.g., the popular quadrature amplitude modulation (QAM).

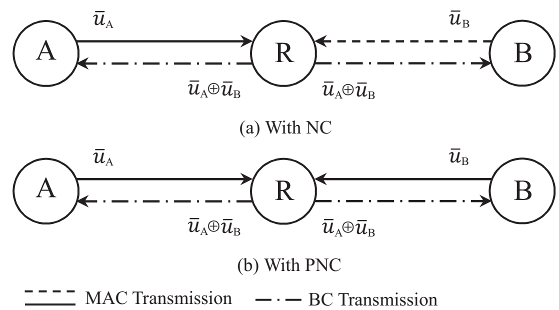


Fig. 1. Message exchange between nodes A and B.

This paper proposes the bit-interleaved coded modulation

(BICM) coded TWR communication systems by employing the energy efficient QAM modulations. In solving the demapping ambiguity problem, iterative decoding or namely the BICM-ID [15] that is performed by R will be introduced. The demapper obtains the channel observations with knowledge of the mapping constellations of both R and the end nodes. By iteratively updating the transmitted symbol pair probabilities, the demapper produces more accurate *a priori* probabilities of the XOR coded bits for the decoder. Both of the TWR Gaussian and fading channels are considered. The proposed work generalizes the iterative demapping-decoding approach for TWR channels. It can also be applied to the scenario without the demapping ambiguity. Our simulation results show that the demapping ambiguity can be overcome by BICM-ID and significant iterative performance gains can be achieved. The iterative decoding performance yields a tendency of approaching the XOR-decoding bound.

II. BICM Coded TWR Communications

Let $\overline{u}_\xi = [u_1^\xi, u_2^\xi, \ldots, u_t^\xi, \ldots, u_l^\xi]$ denote the binary message vector of node ξ, where $\xi \in \{\mathrm{A}, \mathrm{B}\}$ and l is the length of the message vector. A convolutional code of rate r is used to encode the message vectors and the binary codeword of node ξ is $\overline{c}_\xi = [c_1^\xi, c_2^\xi, \ldots, c_{t'}^\xi, \ldots, c_{\frac{l}{r}}^\xi]$. In this paper, the M-ary QAM modulation schemes will be considered and its order is $m = \log_2 M$. In BICM, the serial-to-parallel (S/P) conversion will be performed producing m parallel coded bit sequences $\{c_j^{\xi,1}\}, \{c_j^{\xi,2}\}, \ldots, \{c_j^{\xi,m}\}$, where $j = 1, 2, \ldots, \frac{l}{rm}$. Each sequence will then be randomly interleaved, yielding m interleaved coded bit sequences $\{v_j^{\xi,1}\}, \{v_j^{\xi,2}\}, \ldots, \{v_j^{\xi,m}\}$. It is assumed that both of the end nodes utilize the same interleaving patterns for the m sequences. After interleaving, every m interleaved coded bits constitute vector $\overline{v}_j^\xi = [v_j^{\xi,1}, v_j^{\xi,2}, \ldots, v_j^{\xi,m}]$ which will be mapped to a transmitted symbol of node ξ by

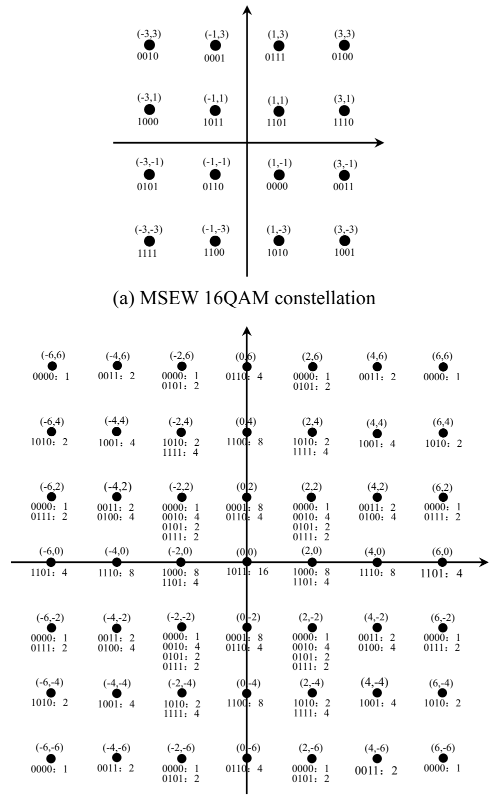
$$x_j^\xi = \Lambda_\xi(\overline{v}_j^\xi), \tag{1}$$

where $\Lambda_\xi(\cdot)$ is the mapping function employed by node ξ. Inversely, $\Lambda_\xi^{-1}(\cdot)$ is the demapping function. Transmitted symbol $x_j^\xi \in \chi_\xi$ and χ_ξ is the signal constellation of node ξ. For simplicity, it is assumed that nodes A and B employ the same mapping scheme such that $\chi_\mathrm{A} = \chi_\mathrm{B}$ and $\Lambda_\mathrm{A}(\cdot) = \Lambda_\mathrm{B}(\cdot)$. However, it is not necessary that A and B have to use the same mapping scheme, e.g., they can employ different types of 16QAM.

The baseband signal model of the TWR channels is

$$y_j = \alpha_j^\mathrm{A} x_j^\mathrm{A} + \alpha_j^\mathrm{B} x_j^\mathrm{B} + n_j, \tag{2}$$

where α_j^A and α_j^B are the fading coefficients experienced by symbols x_j^A and x_j^B, respectively. They are Rayleigh distributed random variables with $\mathbb{E}[(\alpha_j^\mathrm{A})^2] = \mathbb{E}[(\alpha_j^\mathrm{B})^2] = 1$. n_j is the additive white Gaussian noise (AWGN) that is observed by R with noise variance $N_0/2$. When $\alpha_j^\mathrm{A} = \alpha_j^\mathrm{B} = 1$ for all j, it becomes the TWR Gaussian channel. After the MAC transmission, R obtains the received vector $\overline{y} = [y_1, y_2, \ldots, y_{\frac{l}{rm}}] \in \mathbb{R}^{\frac{l}{rm}}$ with which it decodes the XOR message vector of



(a) MSEW 16QAM constellation

(b) Superimposed constellation

Fig. 2. MSEW 16QAM and its superimposed constellations.

$$\overline{u}^\oplus = \overline{u}_\mathrm{A} \oplus \overline{u}_\mathrm{B} = [u_1^\mathrm{A} \oplus u_1^\mathrm{B}, u_2^\mathrm{A} \oplus u_2^\mathrm{B}, \ldots, u_l^\mathrm{A} \oplus u_l^\mathrm{B}]. \tag{3}$$

If it can be correctly decoded, it will be re-encoded and broadcast in the BC stage. This paper focuses on decoding $\overline{u}^\oplus$ with $\overline{y}$.

III. BICM-ID for TWR Channels

In order to iteratively decode the XOR message vector $\overline{u}^\oplus$, we will first visit the superimposed signal constellation that is observed by R. At the meantime, some notations that are used to describe the iterative decoding will be introduced.

A. Superimposed Signal Constellation

As shown by (2), R receives a noisy version of the superimposed signal from nodes A and B. Let s_ϱ^A and s_σ^B denote the constellation points that are chosen from χ_A and χ_B, respectively, where $(\varrho, \sigma) = 1, 2, \ldots, M$. With $\alpha_j^\mathrm{A} = \alpha_j^\mathrm{B} = 1$, the superimposed signal constellation that is observed at R can be defined as:

$$\chi_\oplus = \{s_\tau^\oplus = s_\varrho^\mathrm{A} + s_\sigma^\mathrm{B},\ \forall\ s_\varrho^\mathrm{A} \in \chi_\mathrm{A} \text{ and } s_\sigma^\mathrm{B} \in \chi_\mathrm{B}\}, \tag{4}$$

where $s_\tau^\oplus$ is a superimposed signal of $\chi_\oplus$ with $\tau = 1, 2, \ldots, |\chi_\oplus|$ and $M < |\chi_\oplus| \leq M^2$. For a better illustration, Fig. 2 shows the constellation of the maximum squared Euclidean weight (MSEW) 16QAM [16] and its superimposed constellation under the condition of $\alpha_j^\mathrm{A} = \alpha_j^\mathrm{B} = 1$. With $|\chi_\mathrm{A}| = |\chi_\mathrm{B}| = 16$, $|\chi_\oplus| = 49$. Let $\overline{v}_j^\oplus = [v_j^{\oplus,1}, v_j^{\oplus,2}, \ldots, v_j^{\oplus,m}]$ denote the set of XOR interleaved coded bits, where $v_j^{\oplus,i} = v_j^{\mathrm{A},i} \oplus v_j^{\mathrm{B},i}$ and $i = 1, 2, \ldots, m$. Moreover, we denote $[\overline{v}_j^{\oplus}]_i = v_j^{\oplus,i}$. It can be noticed that a superimposed signal may be demapped to multiple sets of $\overline{v}_j^{\oplus}$. This is because the superposition of different pairs of constellation points that are chosen from χ_{A} and χ_{B} may produce the same superimposed signal point in $\chi_{\oplus}$. For example, let $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ denote a pair of signals that are chosen from the constellation of Fig. 2(a). The superimposed signal $s_\tau^{\oplus}$ of $(-2, 6)$ in Fig. 2(b) is a superposition outcome of the following signal pairs $((-1, 3), (-1, 3))$ whose superposition produces $\overline{v}_j^{\oplus} = [0, 0, 0, 0]$, and $((-3, 3), (1, 3))$ (or $((1, 3), (-3, 3))$) whose superposition produces $\overline{v}_j^{\oplus} = [0, 1, 0, 1]$. In general, $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) \mapsto s_\tau^{\oplus}$ can be a many-to-one map, while $s_\tau^{\oplus} \mapsto (s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ can be a one-to-many map. This causes the demapping ambiguity at R when it tries to decode the XOR message $\overline{u}^{\oplus}$. We aim to overcome such ambiguity by iterative decoding. The following notations that are associated with both the original and the superimposed constellations will be defined.

- Let $\chi_{\oplus}^{i,b}$ denote the set of superimposed constellation points $s_\tau^{\oplus}$ whose demapping produces the ith bit being b and $b \in \{0, 1\}$. If $\Lambda_{\oplus}^{-1}(\cdot)$ is used to denote the demapping function that performs $s_\tau^{\oplus} \mapsto \overline{v}_j^{\oplus}$, $\chi_{\oplus}^{i,b}$ can be defined as:

$$\chi_{\oplus}^{i,b} = \{s_\tau^{\oplus} \in \chi_{\oplus} \mid [\Lambda_{\oplus}^{-1}(s_\tau^{\oplus})]_i = b\}. \quad (5)$$

For example, with $i = 1$ to 4 and $b = 0$ or 1, points of Fig. 2(b) can be categorized into eight different subsets. However, notice that a superimposed signal point can belong to both $\chi_{\oplus}^{i,0}$ and $\chi_{\oplus}^{i,1}$. For instance, $(-2, 6) \in \chi_{\oplus}^{2,0}$ and $(-2, 6) \in \chi_{\oplus}^{2,1}$.

- Let $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})_{\oplus}^{i,b}$ denote the set of pairs of original constellation points that are chosen from χ_{A} and χ_{B} such that $s_\varrho^{\mathrm{A}} + s_\sigma^{\mathrm{B}} = s_\tau^{\oplus} \in \chi_{\oplus}^{i,b}$, i.e.,

$$(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})_{\oplus}^{i,b} = \{(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) \mid s_\varrho^{\mathrm{A}} + s_\sigma^{\mathrm{B}} = s_\tau^{\oplus} \in \chi_{\oplus}^{i,b}\}. \quad (6)$$

This definition enables us to trace back to the original constellations employed by nodes A and B, and identify the pairs of original constellation points whose superposition would yield a point in the set $\chi_{\oplus}^{i,b}$. Since in the TWR channels, the transmitted symbols x_j^{A} and x_j^{B} are the constellation points of s_ϱ^{A} and s_σ^{B}, respectively, we call $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ the transmitted symbol pair.

- Let $\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ denote the set of XOR interleaved coded bits $[v_j^{\oplus,1}, v_j^{\oplus,2}, \ldots, v_j^{\oplus,m}]$ that corresponds to the superposition of s_ϱ^{A} and s_σ^{B}, i.e.,

$$\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = \Lambda_{\mathrm{A}}^{-1}(s_\varrho^{\mathrm{A}}) \oplus \Lambda_{\mathrm{B}}^{-1}(s_\sigma^{\mathrm{B}}). \quad (7)$$

With a M-ary QAM, there are M^2 different transmitted symbol pairs $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$, while there are only M different permutations of $[v_j^{\oplus,1}, v_j^{\oplus,2}, \ldots, v_j^{\oplus,m}]$. Therefore, it can be observed that each transmitted symbol pair corresponds to a particular permutation of $[v_j^{\oplus,1}, v_j^{\oplus,2}, \ldots, v_j^{\oplus,m}]$, while each permutation of $[v_j^{\oplus,1}, v_j^{\oplus,2}, \ldots, v_j^{\oplus,m}]$ corresponds to M different transmitted symbol pairs. Furthermore, we denote $[\overline{v}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})]_i = v_j^{\oplus,i}$. For example, in Fig. 2(a) with $s_\varrho^{\mathrm{A}} = (-3, 3)$ and $s_\sigma^{\mathrm{B}} = (1, 3)$, $\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = [0, 1, 0, 1]$ and $[\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})]_2 = 1$.

B. Iterative Decoding

Armed with the above knowledge, we can now introduce the iterative decoding performed by R. In the following, P_a, P_p and P_e will be used to denote the *a priori*, the *a posteriori* and the extrinsic probabilities, respectively. The channel decoding is performed by the maximum *a posteriori* (MAP) algorithm.

With the received vector $\overline{y}$, the demapper obtains the channel observations by

$$\Pr(y_j | s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = \frac{1}{\pi N_0} \exp\Big(-\frac{||y_j - \alpha_j^{\mathrm{A}} s_\varrho^{\mathrm{A}} - \alpha_j^{\mathrm{B}} s_\sigma^{\mathrm{B}}||^2}{N_0}\Big). \quad (8)$$

We assume the demapper has perfect channel state information (CSI) of α_j^{A} and α_j^{B}. The above channel observation implies given a received symbol y_j, the probability of symbols s_ϱ^{A} and s_σ^{B} have been transmitted by nodes A and B. With M-ary modulations employed by both of the end nodes, each received symbol will spin out M^2 channel observations w.r.t. each distinct transmitted symbol pair $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$. They will be kept unchanged during the iterative decoding. With the channel observations, the demapper further determines the *a posteriori* probabilities of the XOR interleaved coded bits $v_j^{\oplus,i}$ by

$$\begin{aligned} P_p(v_j^{\oplus,i} = b | y_j) &= \mathcal{N}_1 \sum_{(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})_{\oplus}^{i,b}} \Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}} | y_j) \\ &= \mathcal{N}_1 \sum_{(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})_{\oplus}^{i,b}} \Pr(y_j | s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) \cdot \Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}), \end{aligned} \quad (9)$$

where $\mathcal{N}_1 = (\sum_{b \in \{0,1\}} P_p(v_j^{\oplus,i} = b | y_j))^{-1}$ is a normalization factor. $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ is the probability of symbols s_ϱ^{A} and s_σ^{B} being transmitted by the two end nodes, which is called the transmitted symbol pair probability. Since message vectors $\overline{u}_{\mathrm{A}}$ and $\overline{u}_{\mathrm{B}}$ are independent, s_ϱ^{A} and s_σ^{B} are also independent, i.e.

$$\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = \Pr(s_\varrho^{\mathrm{A}}) \cdot \Pr(s_\sigma^{\mathrm{B}}). \quad (10)$$

The superposition of symbols s_ϱ^{A} and s_σ^{B} can be demapped to a set of m XOR interleaved coded bits $\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$. We can calculate the probability $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ with knowledge of the m bits. Since each permutation of $[v_j^{\oplus,1}, v_j^{\oplus,2}, \ldots, v_j^{\oplus,m}]$ corresponds to M transmitted symbol pairs and interleaving warrants the m bits of $\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ being independent, probability $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ can be determined by

$$\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = \frac{1}{M} \prod_{i=1}^{m} P_a([\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})]_i), \quad (11)$$

where $P_a([\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})]_i)$ is the *a priori* probabilities of the XOR interleaved coded bits $v_j^{\oplus,i}$. At the beginning of the iterative decoding, no knowledge of $v_j^{\oplus,i}$ is available and $P_a(v_j^{\oplus,i} = 0) = P_a(v_j^{\oplus,i} = 1) = 1/2$, and $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = \frac{1}{M^2}$ for all the transmitted symbol pairs. This is equivalent to claiming all the points in χ_{A} and χ_{B} have equal probability of occurrence such that $\Pr(s_\varrho^{\mathrm{A}}) = \Pr(s_\sigma^{\mathrm{B}}) = \frac{1}{M}$, and $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}}) = \frac{1}{M^2}$. Once the MAP channel decoding has been performed, more information of the XOR interleaved coded bits will be available and they can be utilized to update $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ as in (11).

The extrinsic probabilities of the XOR interleaved coded bits $v_j^{\oplus,i}$ can be further determined by

$$P_e(v_j^{\oplus,i}=b)=\mathcal{N}_2\frac{P_p(v_j^{\oplus,i}=b|\overline{y})}{P_a(v_j^{\oplus,i}=b)}, \quad (12)$$

where $\mathcal{N}_2 = (\sum_{b\in\{0,1\}} P_e(v_j^{\oplus,i} = b))^{-1}$ is a normalization factor. Since $P_a(v_j^{\oplus,i} = b)$ can also be denoted as $P_a([\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})]_i = b)$, and based on (9) and (11), $P_e(v_j^{\oplus,i} = b)$ of (12) can be written as:

$$P_e(v_j^{\oplus,i}=b)= \mathcal{N}_2\frac{1}{M}\sum_{(s_\varrho^{\mathrm{A}},s_\sigma^{\mathrm{B}})_\oplus^{i,b}}\Pr(y_j|s_\varrho^{\mathrm{A}},s_\sigma^{\mathrm{B}})\prod_{\substack{i'=1\\ i'\neq i}}^{m}P_a([\overline{v}^{\oplus}(s_\varrho^{\mathrm{A}},s_\sigma^{\mathrm{B}})]_{i'}). \quad (13)$$

Therefore, to determine the extrinsic probability $P_e(v_j^{\oplus,i} = b)$, the demapper will first employ the superimposed signal constellation to identify the signal points of set $\chi_\oplus^{i,b}$. For example, if the demapper wants to determine $P_e(v_j^{\oplus,1} = 1)$, it needs to identify points of $\chi_\oplus^{1,1}$. Based on each identified signal point of the set, it further traces back to the original constellations that are employed by the two end nodes by computing $\Lambda_{\mathrm{A}}^{-1}(s_\varrho^{\mathrm{A}}) \oplus \Lambda_{\mathrm{B}}^{-1}(s_\sigma^{\mathrm{B}})$ to find out those transmitted symbol pairs whose superposition can produce the ith bit being b. By going through all the points of subset $\chi_\oplus^{i,b}$, the relevant transmitted symbol pairs $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})_\oplus^{i,b}$ are identified. Hence, to determine a probability $P_e(v_j^{\oplus,i} = b)$, the tracing process requires $O(mM^2|\chi_\oplus^{i,b}|)$ binary computations. This can be facilitated by using a match table between superimposed points of $\chi_\oplus^{i,b}$ and the transmitted symbol pairs $(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})_\oplus^{i,b}$. By identifying the transmitted symbol pairs, the demapper can further specify the use of relevant channel observations $\Pr(y_j|s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ and the calculation of the relevant transmitted symbol pair probabilities $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$. Therefore, the superimposed constellation $\chi_\oplus$ is utilized as an intermediate platform for tracing to the original constellations χ_{A} and χ_{B} in finding the relevant transmitted symbol pairs.

The extrinsic probabilities $P_e(v_j^{\oplus,i})$ will then be deinterleaved and parallel-to-serial (P/S) converted accordingly. The outcome will be mapped to the *a priori* probabilities of the XOR coded bits $c_{t'}^{\oplus}$ by

$$P_e(v_j^{\oplus,i}=b)\mapsto P_a(c_{t'}^{\oplus}=b), \quad (14)$$

where $c_{t'}^{\oplus} = c_{t'}^{\mathrm{A}} \oplus c_{t'}^{\mathrm{B}}$ and $t' = 1, 2, \ldots, \frac{l}{r}$. The MAP decoding algorithm further produces the *a posteriori* probabilities of the XOR coded bits $P_p(c_{t'}^{\oplus}|\overline{y})$ and the XOR information bits $P_p(u_t^{\oplus}|\overline{y})$ with $t = 1, 2, \ldots, l$. The extrinsic probabilities of the XOR coded bits can be determined by

$$P_e(c_{t'}^{\oplus}=b)=\mathcal{N}_3\frac{P_p(c_{t'}^{\oplus}=b|\overline{y})}{P_a(c_{t'}^{\oplus}=b)}, \quad (15)$$

where $\mathcal{N}_3 = (\sum_{b\in\{0,1\}} P_e(c_{t'}^{\oplus} = b))^{-1}$ is a normalization factor. Extrinsic probabilities $P_e(c_{t'}^{\oplus})$ will be S/P converted and interleaved accordingly, and further mapped back to the *a priori* probabilities of the XOR interleaved coded bits by

$$P_e(c_{t'}^{\oplus}=b)\mapsto P_a(v_j^{\oplus,i}=b). \quad (16)$$

With the newly updated probabilities $P_a(v_j^{\oplus,i})$, another round of iteration starts with the demapper recalculating the extrinsic probabilities $P_e(v_j^{\oplus,i})$ as in (13). After a certain number of iterations, the iterative system terminates and estimations of bits $u_t^{\oplus}$ will be made with the knowledge of $P_p(u_t^{\oplus}|\overline{y})$.

It is important to emphasize that the above approach has generalized the iterative demapping-decoding approach for TWR channels. Its application is not limited to the scenarios where the demapping ambiguity arises in the superimposed constellation. Based on (9), we can see that with a received symbol y_j and the M-ary QAM employed by both of the end nodes, the demapping ambiguity prevents us from obtaining M^2 distinct channel observations $\Pr(y_j|s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$. For example, if both of the end nodes employ the MSEW 16QAM of Fig.2(a), over the TWR Gaussian channel each received symbol can only spin out 49 distinct channel observations. However, through the iterative decoding in which the channel decoder feeds back a better estimation of the XOR coded bits, more accurate transmitted symbol pair probabilities $\Pr(s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ can be obtained. They can compensate the demapping ambiguity that is reflected on the channel observations. Hence, iterative decoding is a natural course in curing the demapping ambiguity.

IV. Decoding Performance Analysis

This section presents the BICM-ID performance over both the TWR Gaussian and fading channels. Both the cut-set bound and the XOR-decoding bound are given as the theoretical benchmarks for assessing the optimality of the proposed scheme. Note that the XOR-decoding bound is obtained utilizing the numerical approach of [17].

Figs.3 and 4 shows the frame error rate (FER) performance of BICM-ID over the TWR Gaussian and fading channels, respectively. The rate half 16-state convolutional code with transfer functions $(23, 35)_8$ is employed. Both of the end nodes utilize the MSEW 16QAM modulation. We consider the fast TWR fading channel. Fig.3 shows without iteration, the XOR information cannot be recovered in the TWR Gaussian channel due to the demapping ambiguity. While over the TWR fading channel, the ambiguity can be offset by the multiplying the transmitted symbols x_j^{A} and x_j^{B} with different fading coefficients. That says with $\alpha_j^{\mathrm{A}} \neq \alpha_j^{\mathrm{B}}$, M^2 distinct channel observations $\Pr(y_j|s_\varrho^{\mathrm{A}}, s_\sigma^{\mathrm{B}})$ can be obtained and the XOR information can still be decoded without iteration, but at the cost of a high SNR. By performing iterative decoding, the demapping ambiguity problem is gradually overcome by better estimating the transmitted symbol pair probabilities, improving the iterative decoding performance. More performance enhancement can be made by increasing the iteration number. For example, by increasing the iteration number from two to ten, 3dB performance gain at the FER of 10^{-4} is rewarded over the TWR Gaussian channel. The use of other 16QAM by the two end nodes has also been considered, e.g., the modified set-partitioning (MSP) 16QAM of [15]. We can see

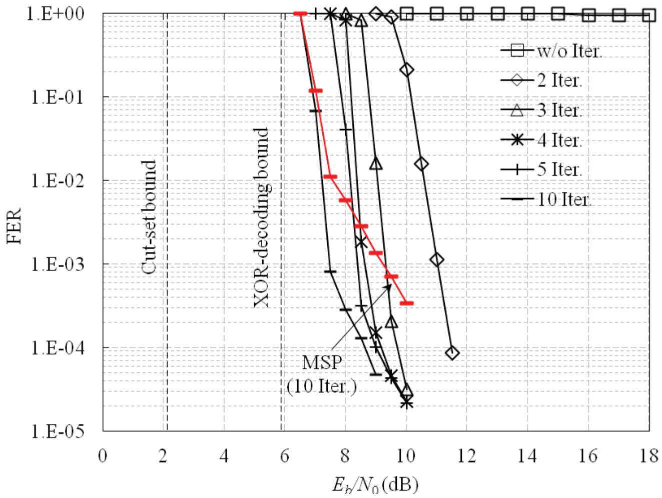


Fig. 3. Iterative decoding performance over the TWR Gaussian channel.

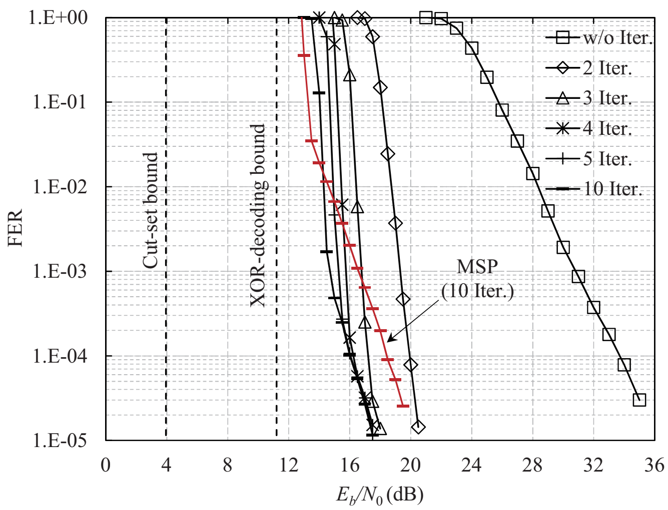


Fig. 4. Iterative decoding performance over the TWR fading channel.

that MSEW 16QAM outperforms MSP 16QAM over both the TWR Gaussian and fading channels.

Our simulation results also show the iterative decoding performance can approach the XOR- decoding bound with a 2.5dB and 4.5dB gap over the TWR Gaussian and fading channels, respectively. This gap is due to the proposed approach still falls short in obtaining individual knowledge of $\Pr(s_\varrho^{\rm A})$ and $\Pr(s_\sigma^{\rm B})$, preventing a more accurate estimation of $\Pr(s_\varrho^{\rm A}, s_\sigma^{\rm B})$ as in (10). Therefore, pursuing a better estimation of $\Pr(s_\varrho^{\rm A}, s_\sigma^{\rm B})$ will be the key in narrowing the gap. This will be the direction of our future endeavor.

V. Conclusion

This paper has proposed a general iterative demapping-decoding approach for TWR channels in realizing spectrally efficient message exchange between two end nodes. It has been shown that iterative decoding is a natural solution to R's demapping ambiguity triggered by employing high order QAMs. With the channel decoding feedback, the demapper can better estimate the transmitted symbol pair probabilities. Consequently, better *a priori* probabilities of the XOR coded bits can be supplied to the channel decoder. It can compensate the demapping ambiguity that arises over the TWR Gaussian channel. Our simulation results have shown that significant iterative decoding gains can be achieved over both the TWR Gaussian and fading channels. The tendency of approaching the XOR-decoding bound over both channels has been demonstrated.

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