A Progressive Chase Type List Decoding Algorithm for Reed-Solomon Codes

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***Abstract*—In this paper, a progressive Chase type list decoding algorithm for Reed-Solomon (RS) codes is proposed. The decoding starts with constructing a set of 2^η ($\eta > 0$) test-vectors which will be arranged in a descending order according to their potential of yielding the intended message. Interpolation and factorization will be performed based on the ordered test-vectors. It is shown significant performance improvement can be achieved over Koetter-Vardy (KV) algorithm with a reduced complexity.**

I. Introduction

We consider an (n, k) Reed-Solomon (RS) code defined in the finite field $\mathbb{F}_q$, and $n = q - 1$. Given a message vector $\mathbf{m} = (m_0\ m_1 \cdots\ m_{k-1}) \in \mathbb{F}_q^k$, the corresponding message polynomial is $m(x) = \sum_{u=0}^{k-1} m_u x^u$. Let $\{\alpha_0, \alpha_1, \ldots, \alpha_{n-1}\} \in \mathbb{F}_q \setminus \{0\}$, the codeword is generated by $\mathbf{c} = (c_0\ c_1\ \cdots\ c_{n-1}) = (m(\alpha_0)\ m(\alpha_1)\ \cdots\ m(\alpha_{n-1})) \in \mathbb{F}_q^n$. In the Chase type list decoding (CLD) algorithm [1], the reliability matrix $\mathbf{\Pi} \in \mathbb{R}_{q \times n}$ will be given as the soft information observed from the channel. Let $\pi_{\nu j}$ denote the entry of $\mathbf{\Pi}$, indicating $\Pr[c_j = \rho_\nu]$, and $\rho_\nu \in \mathbb{F}_q$. Let y_j^1 and y_j^2 denote the most likely and the second most likely symbols for c_j. We define $\gamma_j = \frac{\Pr[c_j = y_j^2]}{\Pr[c_j = y_j^1]}$ as the reliability of the jth received symbol. With $\gamma_j \to 1$, the symbol is less reliable and the η ($\eta > 0$) least reliable symbols will be identified. We define $\Phi = \{\phi_1, \phi_2, \ldots, \phi_\eta\}$ as the set of those symbols' indices, and $\overline{\Phi} = \{0, 1, \ldots, n-1\} \setminus \Phi$. The test-set is constructed as the set of all vectors $\mathbf{y}_i = (y_{i,0}\ y_{i,1}\ \cdots\ y_{i,n-1})$, where $i = 0, 1, \ldots, 2^\eta - 1$. $y_{i,j} = y_j^1$, if $j \in \overline{\Phi}$, and $y_{i,j} \in \{y_j^1, y_j^2\}$, if $j \in \Phi$. Moreover, the k most reliable symbols will be selected to perform the re-encoding transform. For the 2^η test-vectors, 2^η bivariate polynomials $Q_i(x, y)$ will be found by interpolation which is performed based on the re-encoding outcome, forming a polynomial set $\{Q_0(x, y), Q_1(x, y), \ldots, Q_{2^\eta - 1}(x, y)\}$. After factorizing all these polynomials, at most 2^η message vectors can be attained and the one that corresponds to the most likely codeword will be selected. In order to facilitate the decoding, the CLD can be performed in a progressive manner [2].

II. Progressive Chase Type List Decoding

For the n received symbols of each test-vector, n corresponding reliability entries can be picked up from each column of $\mathbf{\Pi}$. In the progressive CLD (PCLD) algorithm, all the test-vectors will be arranged in a descending order according to their potential of yielding the intended message and the potential can be assessed by the following function $\Omega_i = \prod_{j=0}^{n-1} \{\pi_{\nu j} | y_{i,j} = \rho_\nu\}$. If a test-vector $\mathbf{y}_i$ has larger Ω_i value, it is more reliable and should be guaranteed a priority to be decoded. Hence, interpolation and factorization will be performed for the test-vectors according to the above order. Once a decoded codeword satisfies the maximum likelihood (ML) criterion, the decoding will be terminated. Otherwise, the most likely codeword will again be selected.

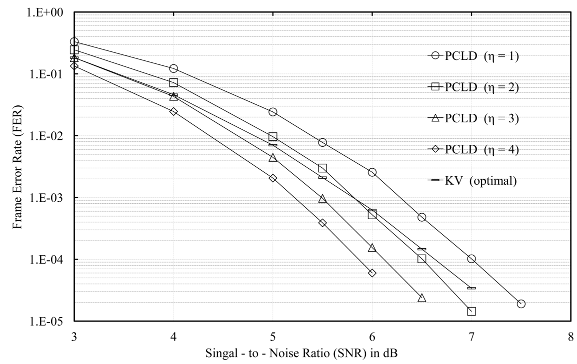

Fig. 1. PCLD Performance of the (15, 11) RS code over the AWGN channel.

TABLE I
Average Complexity For Decoding the (15, 11) RS code.

SNR (dB)	CLD ($\eta = 2$)	PCLD ($\eta = 2$)	CLD ($\eta = 4$)	PCLD ($\eta = 4$)	KV ($l = 3$)
2	8371	7628	26656	23022	67277
4	7867	5018	25222	10785	61492
6	7493	3519	24324	3826	57291
8	7440	3436	24202	3440	56416

III. Simulation Result

Fig.1 shows the performance of the PCLD algorithm in decoding the (15, 11) RS code. It can be seen that with $\eta > 1$, the PCLD algorithm prevails the optimal Koetter-Vardy (KV) decoding performance. Table I shows the average number of finite field arithmetic computations that is required to decode a codeword frame. It shows the PCLD algorithm offers a significant complexity reduction over the CLD and KV (with decoding output list size $l = 3$) algorithms. More importantly, its complexity is channel dependent and can be reduced by increasing the SNR. This is because with the above ordering, a valid output can be delivered at an earlier decoding stage.

References

[1] J. Bellorado and A. Kavcic, "Low-complexity soft-decoding algorithms for Reed-Solomon codes - Part I: an algebraic soft-in hard-out Chase decoder," *IEEE Trans. Inform. Theory*, vol. 56 (3), pp. 945-959, Mar. 2010.

[2] L. Chen, S. Tang and X. Ma, "Progressive algebraic soft-decision decoding of Reed-Solomon codes," *IEEE Trans. Commum.*, vol. 61 (2), pp. 433-442, Feb. 2013.