Assignment of Chapter 6

1. Given a turbo code whose constituent codes are the $(1, 15/13)_8$ recursive systematic convolutional (RSC) codes, its block diagram is shown as in Fig. 1. The interleaving pattern is $\Pi = \{2, 3, 1, 5, 4\}$.

   (a) Please show the state table and trellis for the RSC code.

   (b) Please determine the turbo codeword (without puncturing) of message vector $\boldsymbol{u} = (1, 0, 1, 1, 0)$.

   (c) Please further puncture the above codeword with a puncturing pattern $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

   (d) Transmitting the above punctured turbo codeword through a memoryless channel, the channel observations are presented in Table 1. In the first iteration, please determine the extrinsic probability of BCJR1 ($P_e(u_t)$) and BCJR2 ($P_e(u'_t)$), along with the a posteriori probability of the information bits ($P_p(u_t)$). Estimate the message vector $\hat{\boldsymbol{u}}$.

Solution tip 1: In both the encoding and decoding, you may discard the need of bit tailing. In the BCJR decoders, you can assume that the ending states of the two RSCs are known. E.g., after encoding, if the state of the first RSC is 100, you can assume that this is known by the first BCJR decoder.

Notation 1: Given the binary message vector as $\boldsymbol{u} = (u_1, u_2, ..., u_k)$, the output of the turbo encoder should be $\boldsymbol{c} = \left(u_1, p_1^{(j)}, u_2, p_2^{(j)}, ..., u_k, p_k^{(j)}\right), j = \{1, 2\}$, where the value of j depends on the puncturing pattern P.

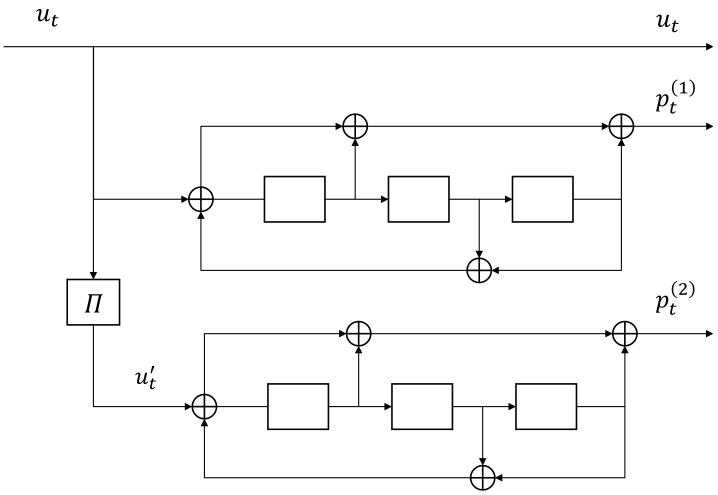

Figure 1: The block dragram of the turbo encoder.

Table 1: The channel observations of codewords, where $j = 1, 2$.

index (t)	1	2	3	4	5
$P_{\text{ch}}(u_t = 0)$	0.11	0.94	0.02	0.34	0.43
$P_{\text{ch}}(u_t = 1)$	0.89	0.06	0.98	0.66	0.57
$P_{\text{ch}}\left(p_t^{(j)} = 0\right)$	0.93	0.29	0.50	0.30	0.80
$P_{\text{ch}}\left(p_t^{(j)} = 1\right)$	0.07	0.71	0.50	0.70	0.20

2. Given a serially concatenated convolutional code (SCCC) whose constituent codes are the $(1, 5/7)_8$ convolutional codes, the block diagram of the SCCC is shown as in Fig. 2. The interleaving pattern is $\Pi = \{8, 3, 7, 6, 9, 1, 10, 5, 2, 4\}$.

   (a) Please determine the SCCC codeword of message vector $\boldsymbol{u} = (0, 0, 1, 1, 1)$ without puncturing.

   (b) Please draw the turbo decoding dragram of the SCCC and describe the decoding process.

   Solution tip 2: For (b), your should explain how the extrinsic and priori probabilities exchange between the two BCJR decoders.

   Notation 2: Please refer to Table 2 for symbol definitions in your description.

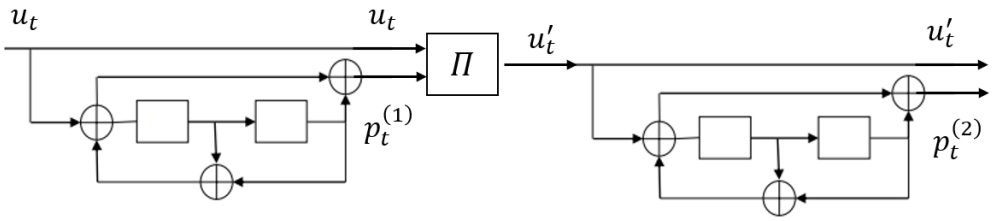

Figure 2: The block dragram of the SCCC encoder.

Table 2: The symbol definition for the SCCC decoder.

Definition	BCJR1	BCJR2
a priori probability	$P_a(u'_t)$	$P_a(u_t)$
a posteriori probability	$P_p(u'_t)$	$P_p(u_t)$
extrinsic probability	$P_e(u'_t)$	$P_e(u_t)$