Assignment of Chapter 5

1. Given an 8-state $(15, 13)_8$ convolutional code, please
   (a) Show its shift register encoder structure.
   (b) Show its state table and trellis. Determine the free distance of this code.
   (c) Determine the codeword of a message vector $\boldsymbol{u} = (1, 1, 1, 0, 1, 0, 1)$. (You should consider adding tailing bits)
   (d) Assume the length of message is 7 bits. Determine the generator matrix of this code. (You should consider adding tailing bits)

2. Given a 4-state $(7, 5)_8$ convolutional code and a received word of

$$\boldsymbol{r} = (11, 01, 11, 00, 00, 01, 11, 00, 10),$$

   please utilize the Viterbi trellis shown in Figure 1 to decode the received word. (The decoding computation can be performed in Figure 1).

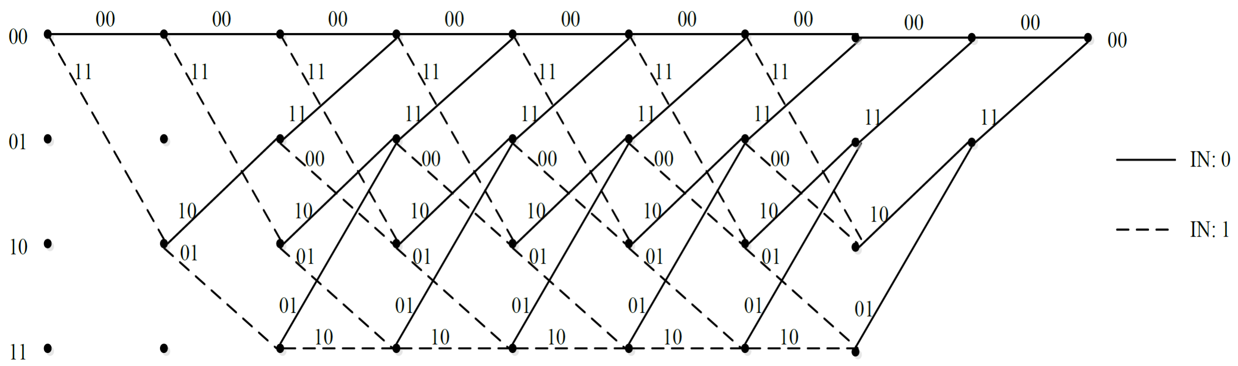

Figure 1: Viterbi trellis of the $(7, 5)_8$ convolutional code

3. Given a rate 2/3 TCM code whose encoder structure and mapping scheme are shown in Figure 2, please

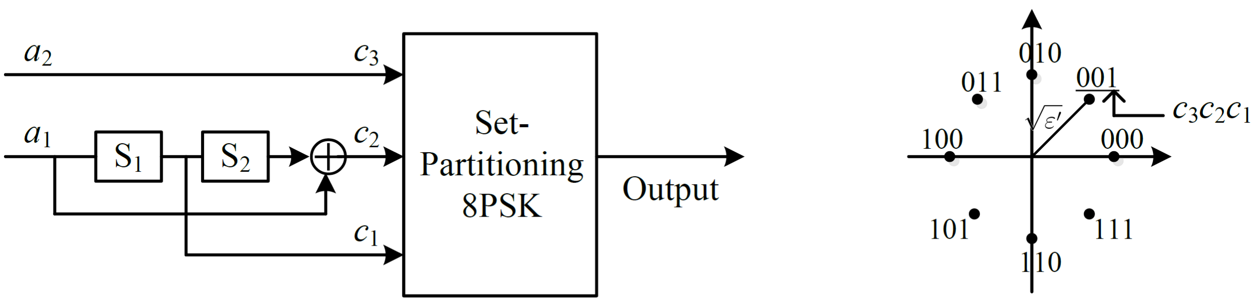

Figure 2: Rate 2/3 TCM encoder and the 8PSK mapping scheme.

(a) Determine trellis of the TCM code.
(b) Determine squared free distance d^2_{free} of the TCM code.
(c) Determine the codeword of a message vector $\boldsymbol{u} = (10, 11, 00, 10)$. (Each pair of bits is in order of (a_1, a_2). You should consider adding tailing bits. Assume $a_2 = 0$ in the tailing bits)
(d) Determine the asymptotic coding gain (in dB) that the TCM coded system is able to achieve over an uncoded QPSK system.

4. Given a trellis of 4-state $(7, 5)_8$ convolutional code shown in Figure 3, we have

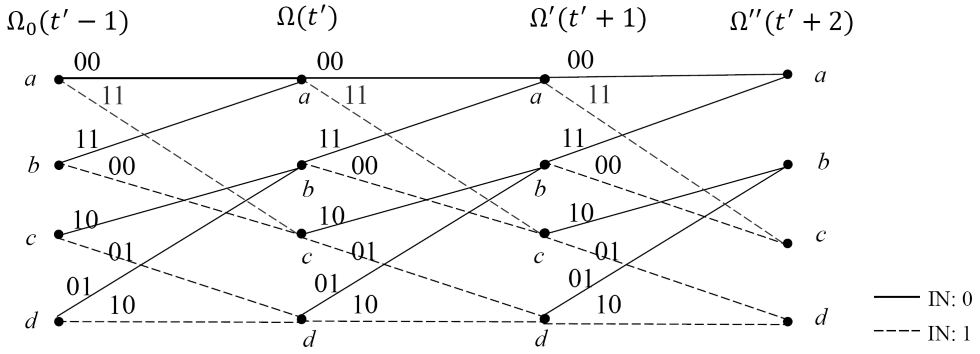

Figure 3: Viterbi trellis of the 4-state $(7, 5)_8$ convolutional code

the channel observations as

$$\begin{cases} P_{\text{ch}}(c^1_{t'-1} = 0) = 0.6 \\ P_{\text{ch}}(c^1_{t'-1} = 1) = 0.2 \end{cases} \begin{cases} P_{\text{ch}}(c^2_{t'-1} = 0) = 0.4 \\ P_{\text{ch}}(c^2_{t'-1} = 1) = 0.4 \end{cases} \quad \begin{cases} P_{\text{ch}}(c^1_{t'} = 0) = 0.3 \\ P_{\text{ch}}(c^1_{t'} = 1) = 0.5 \end{cases} \begin{cases} P_{\text{ch}}(c^2_{t'} = 0) = 0.1 \\ P_{\text{ch}}(c^2_{t'} = 1) = 0.7 \end{cases} \quad \begin{cases} P_{\text{ch}}(c^1_{t'+1} = 0) = 0.4 \\ P_{\text{ch}}(c^1_{t'+1} = 1) = 0.4 \end{cases} \begin{cases} P_{\text{ch}}(c^2_{t'+1} = 0) = 0.7 \\ P_{\text{ch}}(c^2_{t'+1} = 1) = 0.1 \end{cases},$$

(In continuous channel, channel observations are the probability densities. Hence their sum may not necessarily be equal to 1)
the probabilities of the beginning states at time instant $t' - 1$ are

$$A_{t'-1}(a) = 0.3, \quad A_{t'-1}(b) = 0.3, \quad A_{t'-1}(c) = 0.2, \quad A_{t'-1}(d) = 0.2$$

and the probabilities of the ending states at time instant $t' + 2$ are

$$B_{t'+2}(a) = 0.6, \quad B_{t'+2}(b) = 0.1, \quad B_{t'+2}(c) = 0.2, \quad B_{t'+2}(d) = 0.1 \ \cdot$$

(a) Determine the state transition probabilities for all transitions. (Assume we do not have knowledge of information bits)
(b) Determine the probability of each beginning state at time instant t'. (That is to determine $A_{t'}(a), A_{t'}(b), A_{t'}(c), A_{t'}(d)$)
(c) Determine the probability of each ending state at time instant $t' + 1$. (That is to determine $B_{t'+1}(a), B_{t'+1}(b), B_{t'+1}(c), B_{t'+1}(d)$)
(d) Determine the a posteriori probability of information bit at time instant t'. Estimate this information bit.