Assignment of Chapter 6

1. Consider a turbo code whose constituent codes are the $(1, 15/13)_8$ recursive systematic convolutional (RSC) codes, its block diagram is shown as in Fig. 1. The interleaving pattern is $\Pi = \{2, 3, 1, 5, 4\}$. Please

   (a) Determine the state table and trellis for the RSC code.

   (b) Determine the turbo codeword (without puncturing) of message vector $\boldsymbol{u} = (1, 0, 1, 1, 0)$.

   (c) Puncture the above codeword with a puncturing pattern $\mathbf{P} = \left[\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right]$.

   (d) Transmitting the above punctured turbo codeword through a memoryless channel, the channel observations are presented in Table 1. In the first iteration, please determine the *extrinsic* probabilities of decoder BCJR-1 (i.e., $P_e(u_t)$) and decoder BCJR-2 (i.e., $P_e(u'_t)$), along with the *a posteriori* probability of the information bits (i.e., $P_p(u_t)$). Meanwhile, please estimate the message vector $\hat{\boldsymbol{u}}$ based on $P_p(u_t)$.

   Solution tip 1: In both the encoding and decoding, you may discard the need of bit tailing. In the BCJR decoders, you can assume that the ending states of the two RSCs are known. E.g., after encoding, if the state of the first RSC is "100", you can assume that this is known by the first BCJR decoder.

   Notation 1: Given the binary message vector as $\boldsymbol{u} = (u_1, u_2, ..., u_k)$, the output of the turbo encoder should be $\boldsymbol{c} = \left(u_1, p_1^{(j)}, u_2, p_2^{(j)}, ..., u_k, p_k^{(j)}\right)$, $j \in \{1, 2\}$, where the value of j depends on the puncturing pattern P.

   Notation 2: For simplicity, we use BCJR-1 and BCJR-2 to denote the fist and the second BCJR decoders in the turbo decoding, respectively.

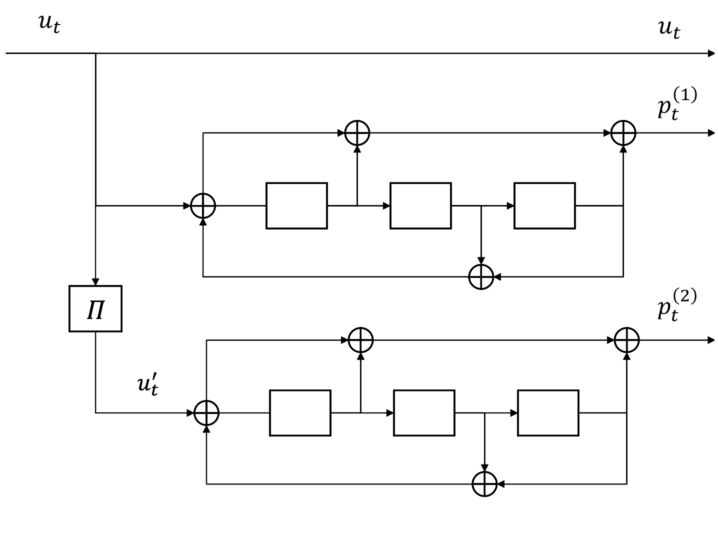

Figure 1: The block dragram of the turbo encoder.

Table 1: The channel observations of codewords, where $j \in 1, 2$.

Index (t)	1	2	3	4	5
$P_{\text{ch}}(u_t = 0)$	0.11	0.94	0.02	0.34	0.43
$P_{\text{ch}}(u_t = 1)$	0.89	0.06	0.98	0.66	0.57
$P_{\text{ch}}\left(p_t^{(j)} = 0\right)$	0.93	0.29	0.50	0.30	0.80
$P_{\text{ch}}\left(p_t^{(j)} = 1\right)$	0.07	0.71	0.50	0.70	0.20

2. Consider a serially concatenated convolutional (SCC) code whose constituent codes are the $(1, 5/7)_8$ convolutional codes, the block diagram of the SCC code is shown as in Fig. 2. The interleaving pattern is $\Pi = \{8, 3, 7, 6, 9, 1, 10, 5, 2, 4\}$. Please

   (a) Determine the codeword of this SCC code with message vector $\boldsymbol{u} = (0, 0, 1, 1, 1)$ and its code rate. Note that the codeword need not be punctured.

   (b) Draw the turbo decoding dragram of the SCC code, and describe the decoding process.

   Solution tip 2: For (b), your should explain how the *extrinsic* and the *a priori* probabilities exchange between the two BCJR decoders.

   Notation 3: Please refer to Table 2 for symbol definitions in your description.

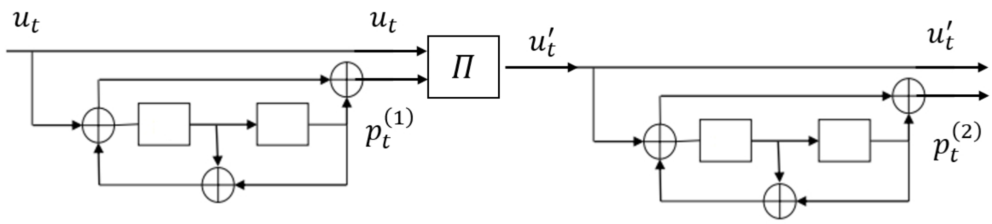

Figure 2: The block dragram of the SCC encoder.

Table 2: The symbol definition of the SCC decoder.

Definition	BCJR-1	BCJR-2
A priori prob.	$P_a(u'_t)$	$P_a(u_t)$
A posteriori prob.	$P_p(u'_t)$	$P_p(u_t)$
Extrinsic prob.	$P_e(u'_t)$	$P_e(u_t)$