Assignment of Chapter 4

1. Given a (15, 11) systematic Hamming code, its generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&1&1&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&1&1&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&1&0&0&0&0&0&0&0&1&1&0&1 \\ 0&0&0&0&1&0&0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&1&0&0&0&0&1&1&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&0&1&0&1&0&1&1 \\ 0&0&0&0&0&0&0&0&0&0&1&1&0&0&1 \end{bmatrix}.$$

(1) Please determine its parity-check matrix.

(2) Please determine whether the received word (1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0) is a valid codeword. If not, please decode it.

(3) Given the received word (?, 0, 0, 0, 1, 1, 0, 1, 0, 1, ?, 0, 0, 1, 1), where "?" denotes an erasure, compute the decoded codeword and determine the maximum number of erasures that this code is guaranteed to correct.

2. Let $g(x) = 1 + x^2 + x^3 + x^4$ be the generator polynomial of a (7, 3) cyclic code.

(1) Please determine its parity-check matrix.

(2) Please determine its systematic generator matrix.

(3) Please determine the codebook generated by $g(x)$ and the minimum Hamming distance.

3. Let n and k be integers such that $n \geq k$. Suppose that $\mathbf{G}$ and $\mathbf{H}$, of dimension $k \times n$ and $(n-k) \times n$ respectively, are generator and parity-check matrices for a linear code $C \in \mathbb{F}_2^n$. Is the $n \times n$ matrix $\begin{pmatrix} \mathbf{G} \\ \mathbf{H} \end{pmatrix}$ necessarily invertible? Please prove it or exhibit a counterexample.