

Assignment of Chapter 3

1. Given a random variable set of $X = \{1,2,3,4,5\}$ with probabilities

$$P = \{\frac{1}{8}, \frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}\},$$

- (1) Please calculate the source entropy.
- (2) Please construct its Shannon-Fano code and calculate the expected length.

2. Given a random variable set of $X = \{1,2,3,4,5,6\}$ with probabilities

$$P = \{\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}\},$$

- (1) Please construct a binary Huffman code;
- (2) Please construct a ternary Huffman code;
- (3) Please discuss whether the above two cases are optimal source codes.

3. Given a random variable set of $X = \{x_1, x_2, \dots, x_u\}$ with probabilities

$$P = \{\frac{1}{u}, \frac{1}{u}, \dots, \frac{1}{u}\},$$

its binary Huffman code has codeword lengths $l_1, l_2, \dots, l_u$. Let $l_{\max} = \max l_i$, where $i \in \{1, 2, \dots, u\}$.

- (1) Please prove that $\sum_{i=1}^u 2^{-l_i} = 1$.
- (2) Please prove that $\forall i, l_i = l_{\max}$ or $l_i = l_{\max} - 1$.