Assignment of Chapter 1

1. Given the joint distribution of X and Y, please determine:

$Y \backslash X$	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

   (a) $H(X),\ H(Y)$.
   (b) $H(X|Y),\ H(Y|X)$.
   (c) $H(X,Y)$.
   (d) $I(X;Y)$.

2. Consider the entropy function $H(\underline{p}) = H(p_1, p_2, \ldots, p_u) = -\sum_{i=1}^{u} p_i \log p_i$ defined on the set of u-dimensional probability vectors $\underline{p}$, where $p_i > 0$ and $\sum_{i=1}^{u} p_i = 1$.

   (a) Find the maximum value of $H(\underline{p})$, and determine all probability vectors $\underline{p}$ that achieve this maximum.

   (b) Find the minimum value of $H(\underline{p})$, and determine all probability vectors $\underline{p}$ that achieve this minimum.

3. Using the definition of relative entropy and of conditional probability,
   (a) please prove that:

$$D\big(p(x,y), q(x,y)\big) = D\big(p(x), q(x)\big) + D\big(p(y|x), q(y|x)\big);$$

   (b) please prove that:

$$D\big(p(x,y) \mid q(x,y)\big) \geq D\big(p(x) \mid q(x)\big).$$