Assignment of Chapter 4

1. Consider a (7, 4, 3) Hamming code whose generator matrix is defined as

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

   Please

   (a) Encode the message vector $(1, 0, 0, 1)$ using **G**.

   (b) Determine the parity-check matrix **H**.

   (c) Determine the systematic generator matrix $\mathbf{G}_{\text{sys}}$, and complete the systematic encoder structure as in Figure 1.

   (d) Determine whether the received word $(1, 1, 1, 0, 1, 0, 1)$ is a valid codeword for $\mathbf{G}_{\text{sys}}$. If not, correct it using **H** and give the decoded message bits.

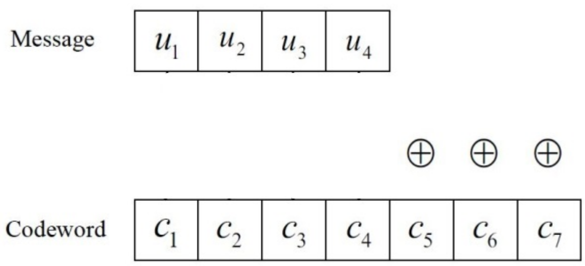

Figure 1: The encoder structure using $\mathbf{G}_{\text{sys}}$.

2. Consider a (7, 3) cyclic code with the generator polynomial of $g(x) = x^4 + x^3 + x^2 + 1$.
   Please

   (a) Determine the generator matrix **G**.

   (b) Determine the parity-check matrix **H**.

   (c) List all codewords generated by $g(x)$, and determine the minimum Hamming distance.

   (d) Determine whether the received word $(1, 0, 1, 1, 1, 0, 1)$ is a valid codeword. If not, please correct it using the maximum likelihood (ML) decoding algorithm.

3. Consider an (8, 5) systematic block code, whose check equations are as follows

$$\begin{cases} p_1 = u_1 + u_2 + u_5, \\ p_2 = u_1 + u_3 + u_5, \\ p_3 = u_1 + u_4 + u_5, \end{cases}$$

where $\{u_1, u_2, u_3, u_4\}$ denote the message bits, and $\{p_1, p_2, p_3\}$ denote the check bits. Please

(a) Determine the generator matrix **G** and the parity-check matrix **H**.

(b) Prove that this block code is a cyclic code.