Assignment of Chapter 1
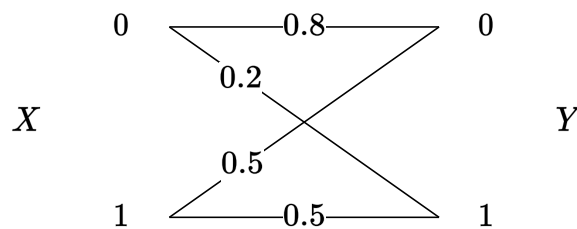
1. Let $\Pr(X, Y)$ be given by

X \ Y	0	1
0	1/4	1/2
1	0	1/4

   Please find

   (a) $H(X)$, $H(Y)$
   (b) $H(X|Y)$, $H(Y|X)$
   (c) $H(X, Y)$
   (d) $H(Y) - H(Y|X)$
   (e) $I(X; Y)$
   (f) Draw a Venn diagram for the above measures in (a) through (e)

2. Given the binary input binary output channel as

   We know $\Pr(X = 0) = 0.8$, $\Pr(X = 1) = 0.2$, $\Pr(Y = 0|X = 0) = 0.8$, $\Pr(Y = 1|X = 0) = 0.2$, $\Pr(Y = 0|X = 1) = 0.5$, and $\Pr(Y = 1|X = 1) = 0.5$. Please determine the mutual information of such a channel.

3. Let X, Y, and Z be joint random variables. Please prove the following inequalities and find conditions for equality.

   (a) $H(X, Y|Z) \geq H(X|Z)$
   (b) $H(X, Y, Z) - H(X, Y) \leq H(X, Z) - H(X)$